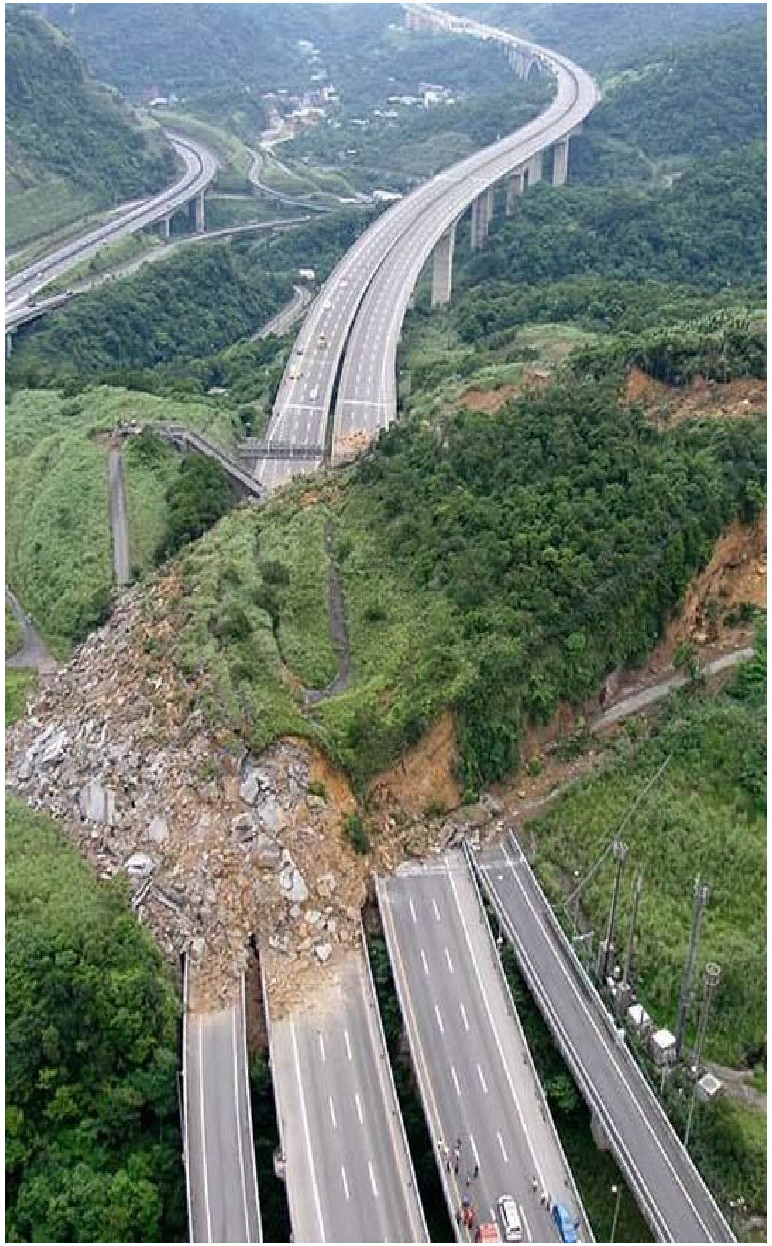

Geomechanics

LECTURE 6

CRITICAL STATE CONCEPT

PROF. LYESSE LALOUI

Laboratory of soil mechanics – Fall 2024

14.10.2023

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Semester

Geomechanics
General Course overview

Basic concepts

Classical constitutive framework

- Elasticity - Linear & Non-linear
- Plasticity Perfect-plasticity
- Hardening elasto-plasticity
- Critical state concept
- Modified Cam-Clay

Practical aspects

- Geomechanics in Practice
- Numerical modelling
- Site visit
- In-situ stress
- Course Project
- Retaining structures

Advanced constitutive models

- Unsaturated behavior
- THM behaviour of geomaterials
- Bio-cementation
- Time-dependent behaviour

Topics

Content

- The critical state concept
- Experimental evidences Conventional Triaxial Compression tests
  - Isotropic drained compression
  - Shearing phase on a NC soil in drained and undrained conditions
  - Shearing phase on a OC soil in drained and undrained conditions
- Critical state and shear strength
- Examples

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- The critical state concept
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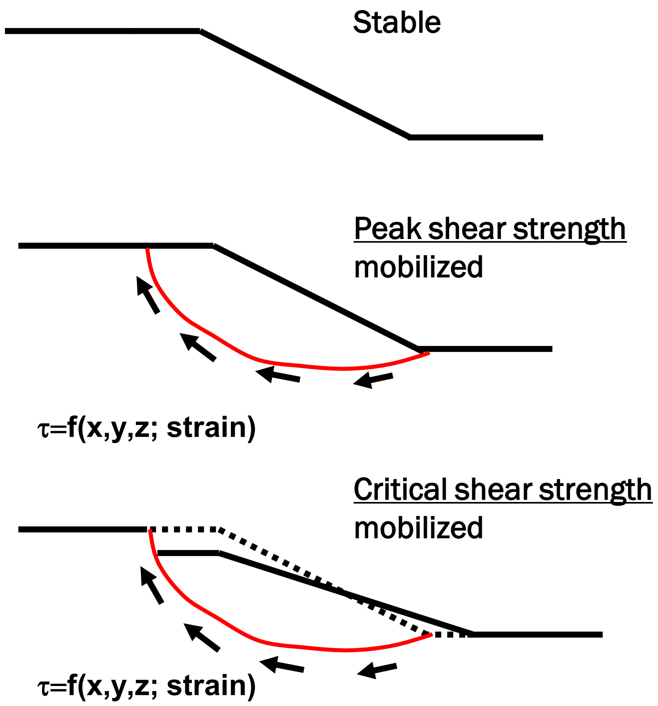
The critical state concept

Practical example: progressive failure in a slope

Belmont/Lausanne landslide 1990

Critical shear strength mobilized (large strains)



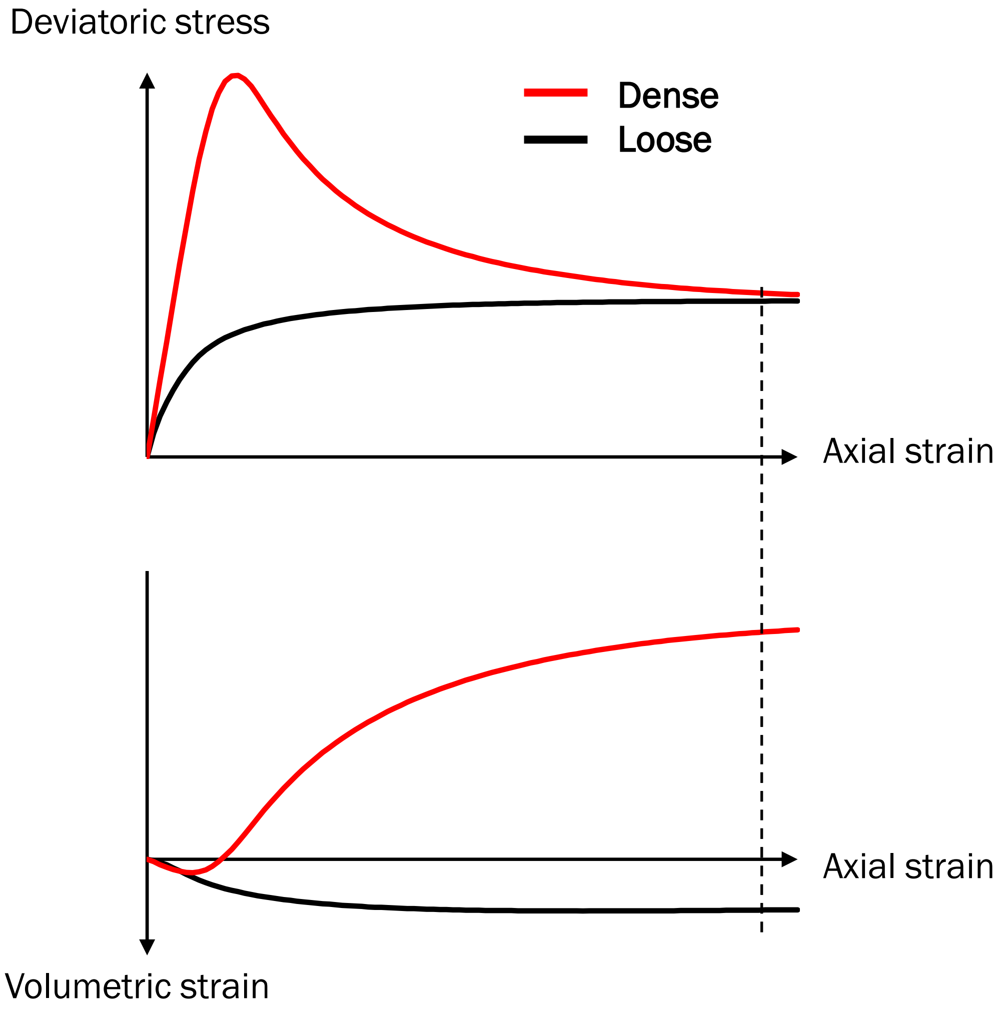
The **shear strength** of a soil depends on the **strain** magnitude

The "**Critical State**" defines the **ultimate shear strength** that the soil can reach at a critical void ratio

The critical state concept

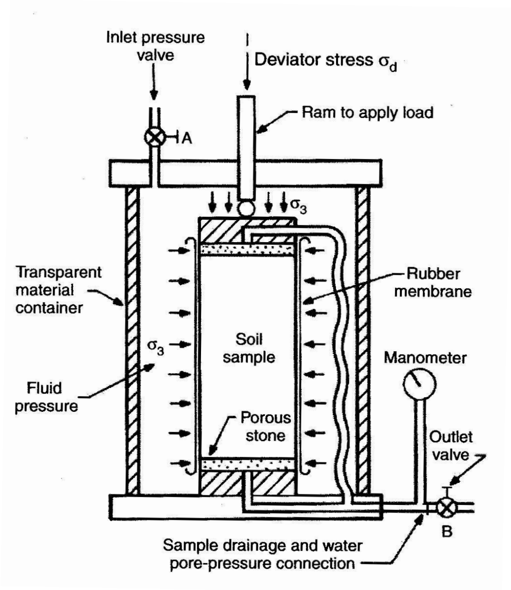
Critical state:

Ultimate state in which shearing can continue without change in volume

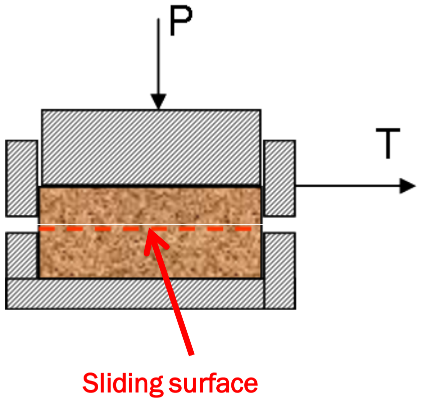


The critical state concept

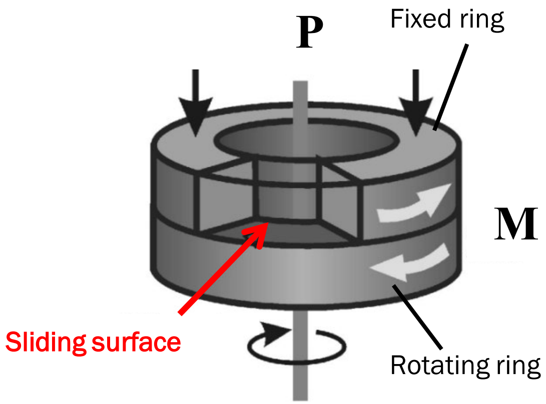
Laboratory tests for analysing the shear strength of a soil



Triaxial test



Direct shear test



Ring shear test

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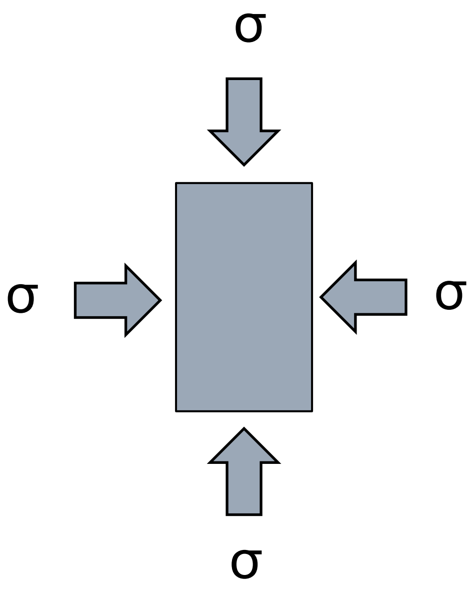
Isotropic drained compression

Principle • Compression in all direction with the same magnitude

Use • Evaluation of the volumetric soil behaviour

Test • First stage of a triaxial test: Consolidation phase



Isotropic stress state

$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma = p$$
$$\sigma'_1 = \sigma'_2 = \sigma'_3 = \sigma' = p'$$
$$q = 0$$

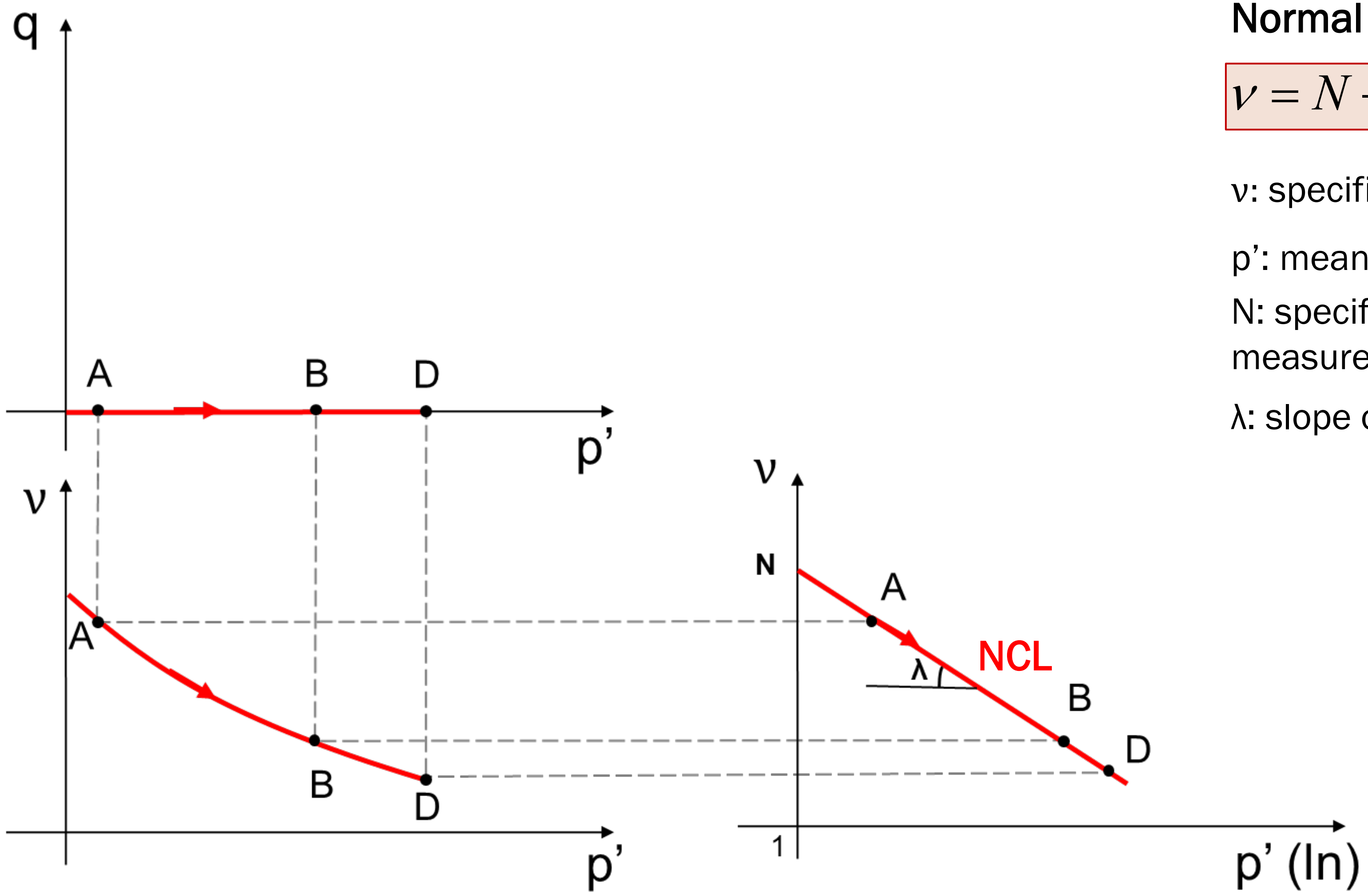
Drained condition

$$\Delta p_w = 0 \quad \Rightarrow \quad \Delta p = \Delta p'$$

If $p_{w,0} = 0 \quad \Rightarrow \quad p = p'$

Isotropic drained compression

Stress path and stress-specific volume relation



Normal Compression Line (NCL)

$$\boxed{v = N - \lambda \ln p'}$$

ν: specific volume (ν =1+e)

p': mean effective stress

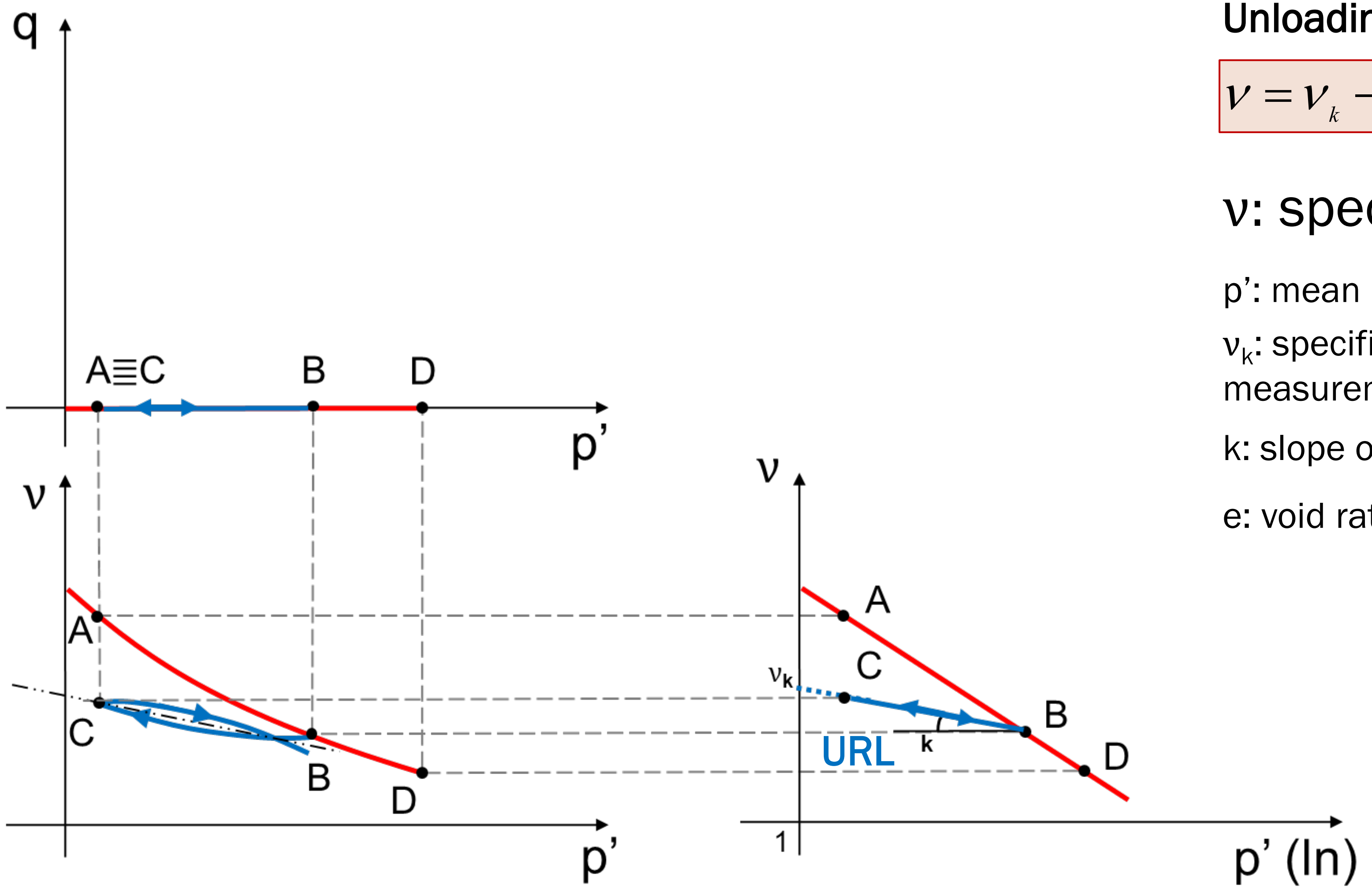
N: specific volume at p'=1 in the adopted measurement system

λ: slope of the NCL (plastic compressibility)

The NCL is unique for a given soil

Isotropic drained compression

Stress path and stress-specific volume relation



Unloading Reloading Line (URL)

$$\boxed{v = v_k - k \ln p'}$$

ν: specific volume ($\nu = 1+e$)

p': mean effective stress

ν_k: specific volume at p'=1 in the adopted measurement system

k: slope of the URL (elastic compressibility)

e: void ratio

Isotropic drained compression

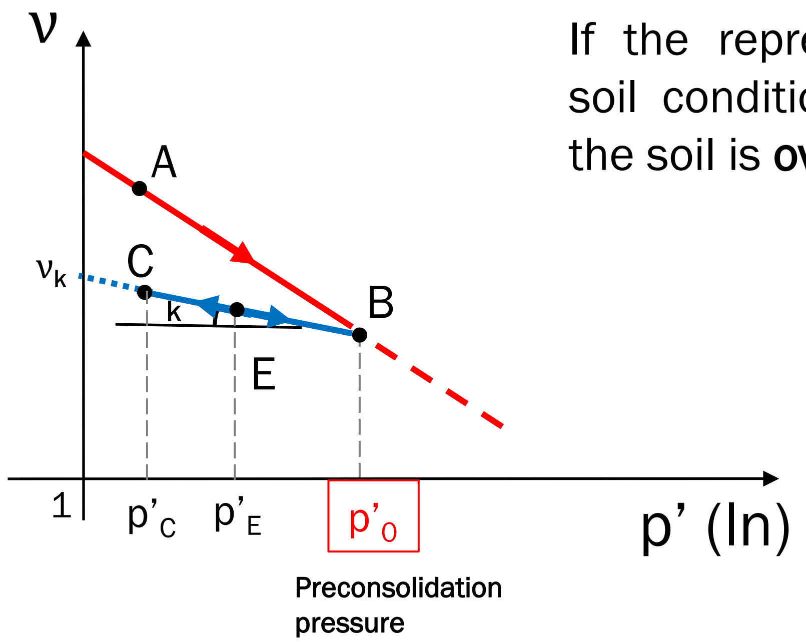
Preconsolidation pressure p'_0 and the Overconsolidation ratio OCR

- The **preconsolidation pressure** (p'$_0$): Maximum mean effective stress to which the soil has been subjected during the past

- The **overconsolidation ratio** (**OCR**) Ratio between the preconsolidation pressure and the actual mean effective stress

$$\boxed{OCR = \frac{p'_0}{p'}}$$

p'$_0$: preconsolidation pressure
p': actual mean effective stress



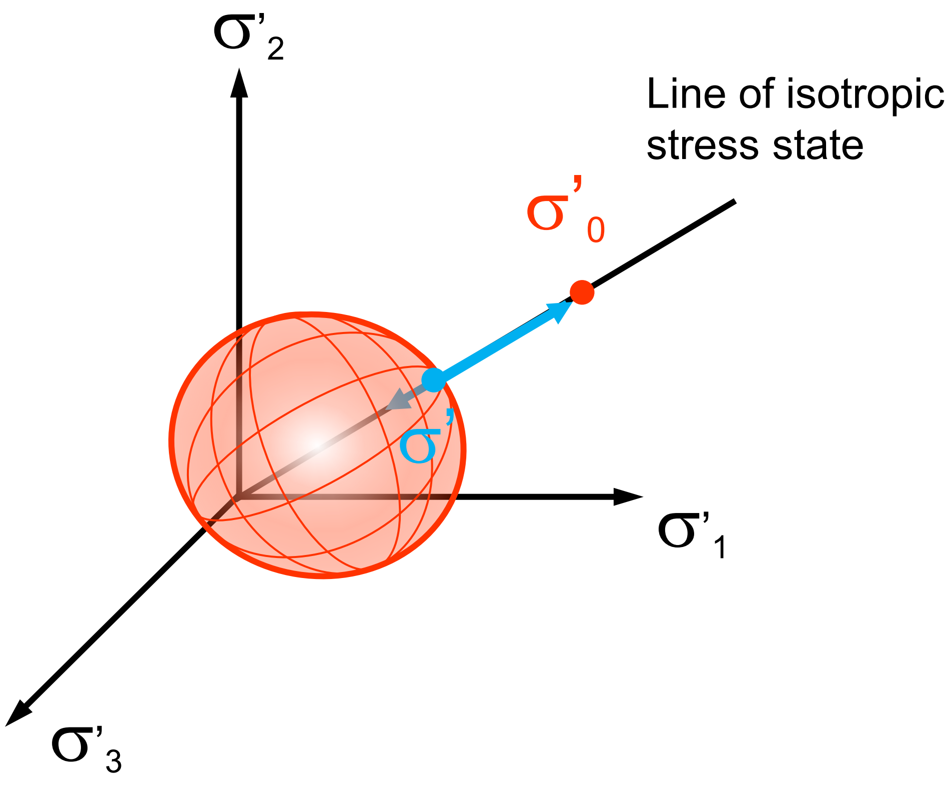
If the representative point of the soil condition belongs to the **URL**, the soil is **overconsolidated** (OCR>1)

Example:

$$OCR_E = \frac{p'_0}{p'_E} > 1 \quad OCR_C = \frac{p'_0}{p'_C} > 1 \quad \text{with} \quad OCR_C > OCR_E$$

Isotropic drained compression

Preconsolidation pressure p'_0 and the Overconsolidation ratio OCR



- Isotropic drained compression:

  The sample is first subjected to an isotropic compression σ'_0 followed by unloading to σ (OCR >1)

  → Increase of the elastic domain

  → Density – dependent behaviour

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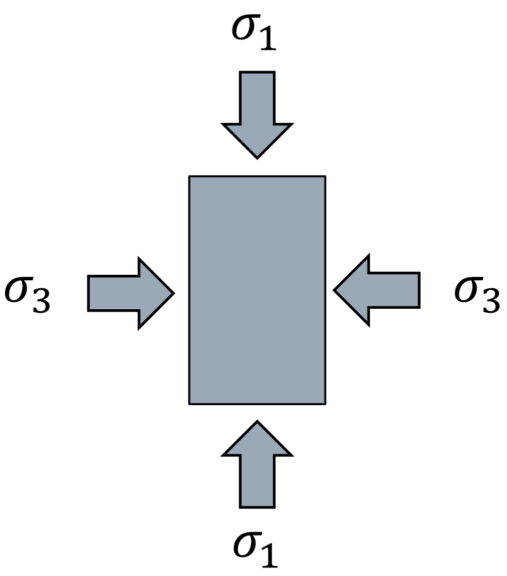
Shearing a NC soil in drained compression

Principle • Increase of the axial stress and imposed drained conditions

Use • Evaluation of the soil shear strength

Test • Second stage of a triaxial test with drainage valves open



Initial state of the soil $OCR = \frac{p'_0}{p'} = 1$ On the NCL

Stress state variables

$$p = \frac{\sigma_1 + 2\sigma_3}{3} \qquad p' = \frac{\sigma'_1 + 2\sigma'_3}{3}$$

$$q = \sigma_1 - \sigma_3$$

Drained condition

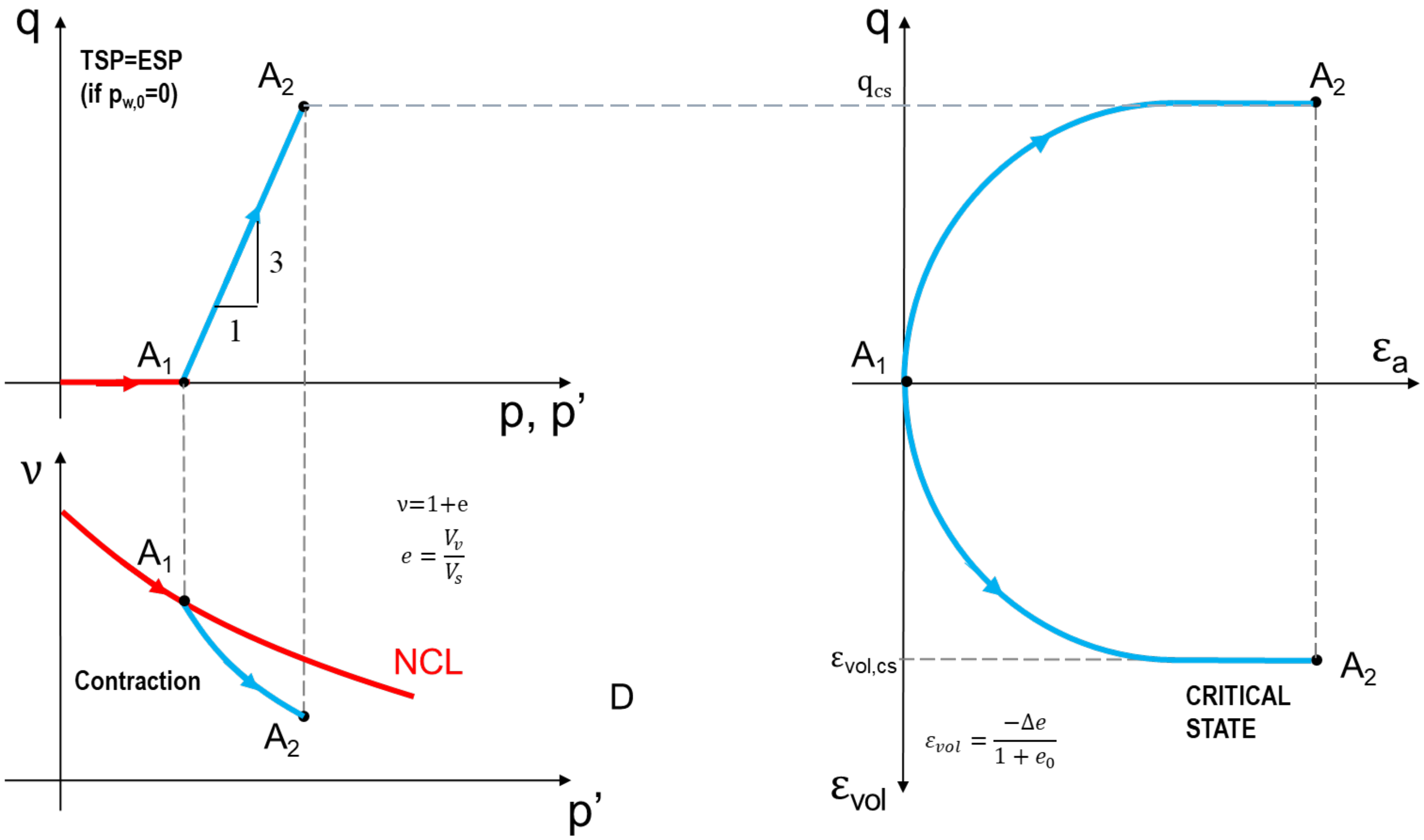
$$\Delta p_w = 0 \quad \Rightarrow \quad \Delta p = \Delta p'$$

If $p_{w,0} = 0 \quad \Rightarrow \quad p = p'$

Shearing a NC soil in drained compression

Stress path and stress-strain response

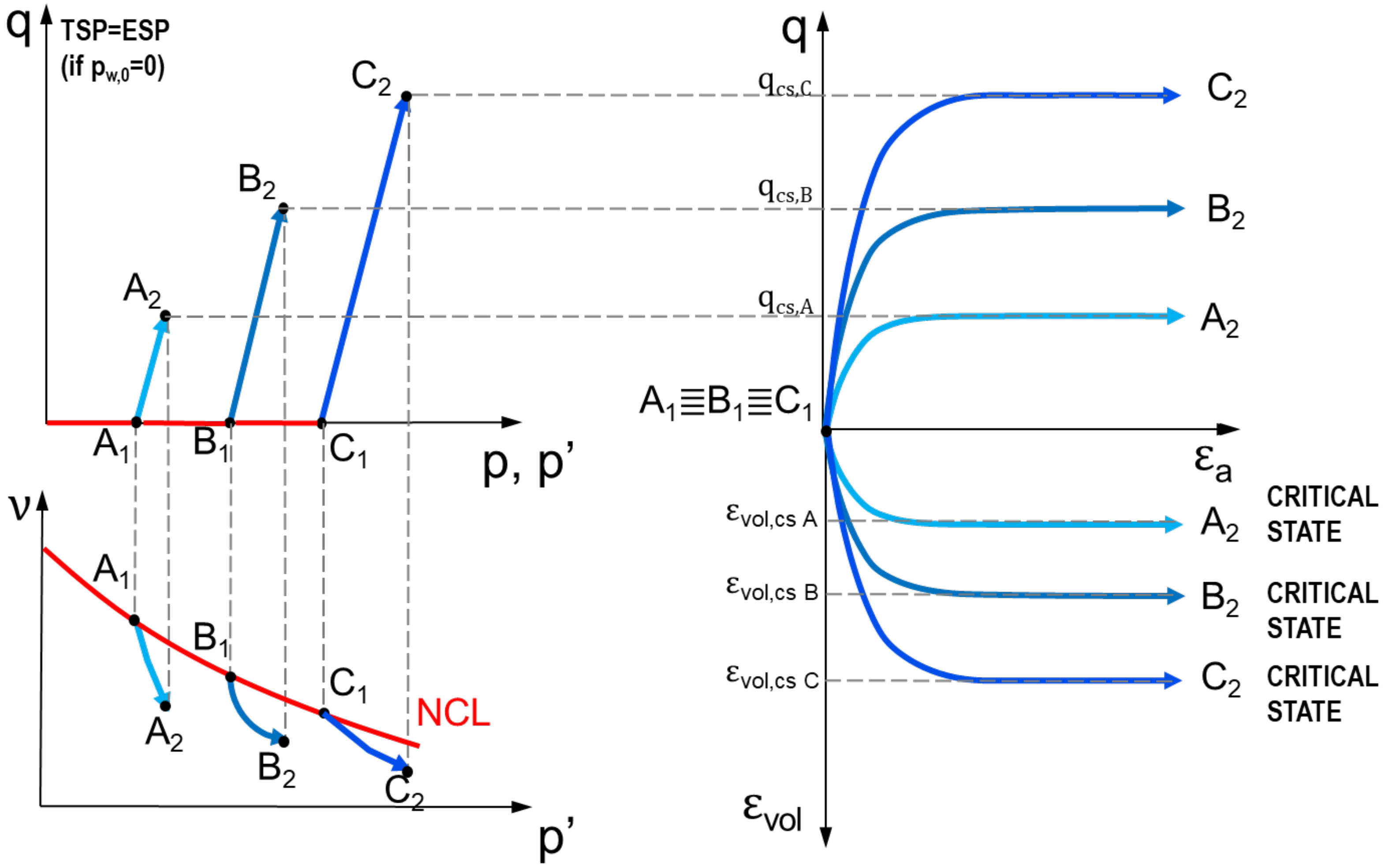
A single sample (A)



Shearing a NC soil in drained compression

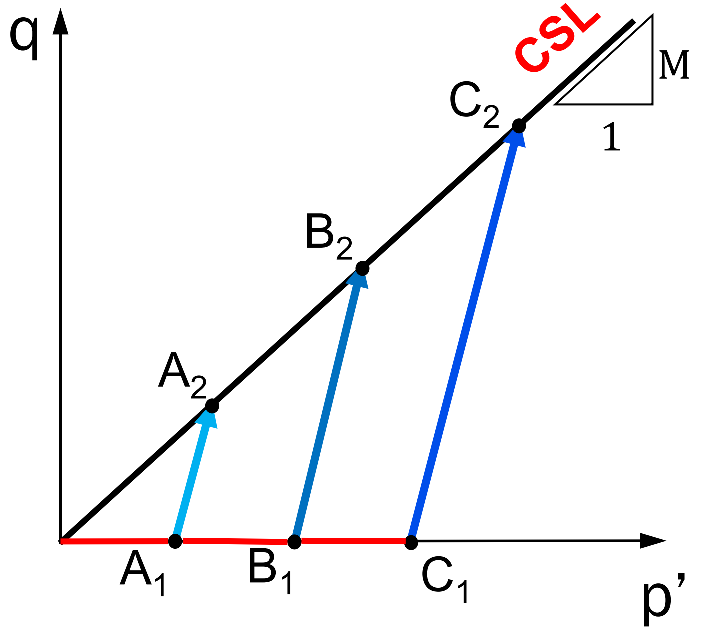
Stress path and stress-strain response

Three samples (A,B,C) consolidated at three different confinement



Shearing a NC soil in drained compression

Critical State Line



In the q - p' plane

All points representative of the soil condition at the **critical state** (A_2,B_2,C_2) in the q-p' plane are well interpretable by a **straight line** passing through the origin

$$q = Mp'$$

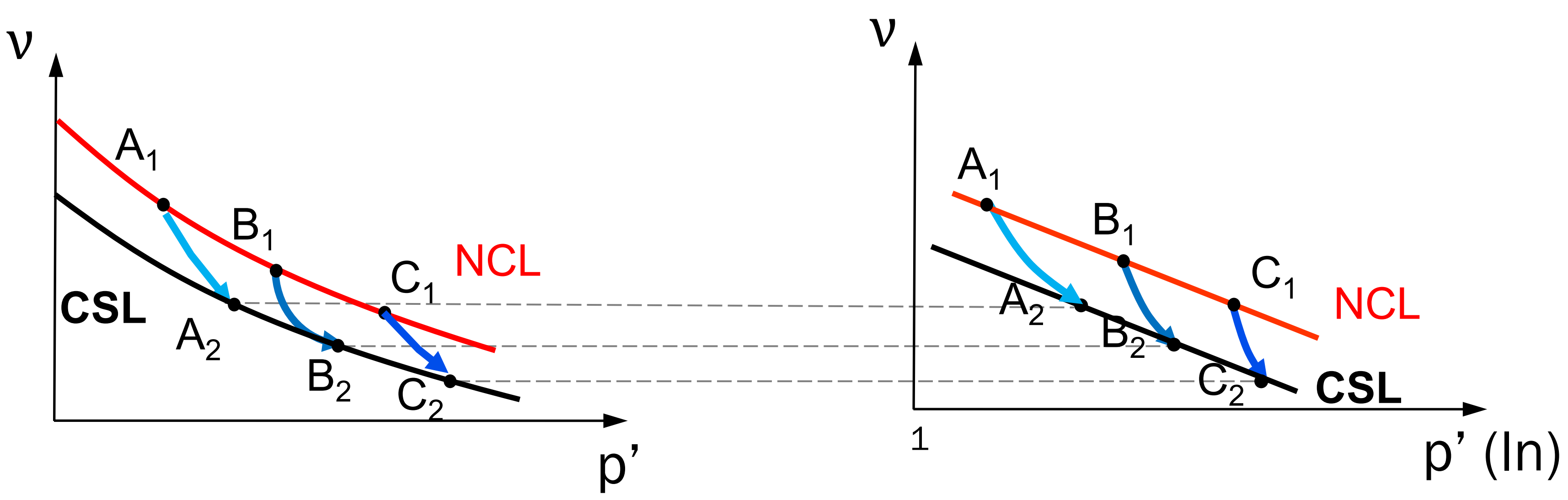
p': Mean effective stress

q: Deviatoric stress

M: Slope of the CSL in the q-p' plane

Shearing a NC soil in drained compression

Critical State Line (CSL) in the ν - p' plane



All points representative of the soil condition at the **critical state** (A_2,B_2,C_2) are well interpretable by a **straight line in the ν-p'(ln) plane** :

$$\nu = \Gamma - \lambda \ln p'$$

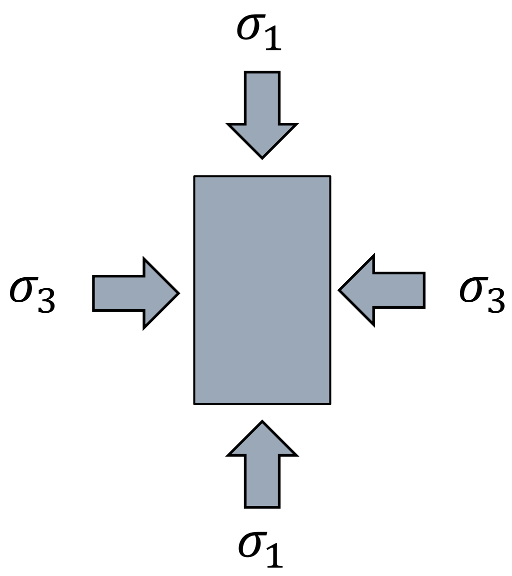
ν : specific volume (ν=1+e)
p': mean effective stress
Γ : specific volume at p'=1 in the adopted measurement system
Λ : slope of the NCL and of the CSL (plastic compressibility)

Shearing a NC soil in undrained compression

Principle • Increase of the axial stress and imposed undrained conditions

Use • Evaluation of the soil shear strength

Test • Second stage of a triaxial test with drainage valves closed



Initial state of the soil $OCR = \frac{p'_0}{p'} = 1$ On the NCL

Stress state variables

$$p = \frac{\sigma_1 + 2\sigma_3}{3} \qquad p' = \frac{\sigma'_1 + 2\sigma'_3}{3}$$

$$q = \sigma_1 - \sigma_3$$

Undrained condition

$$\Delta p_w \neq 0 \Rightarrow \Delta p \neq \Delta p'$$

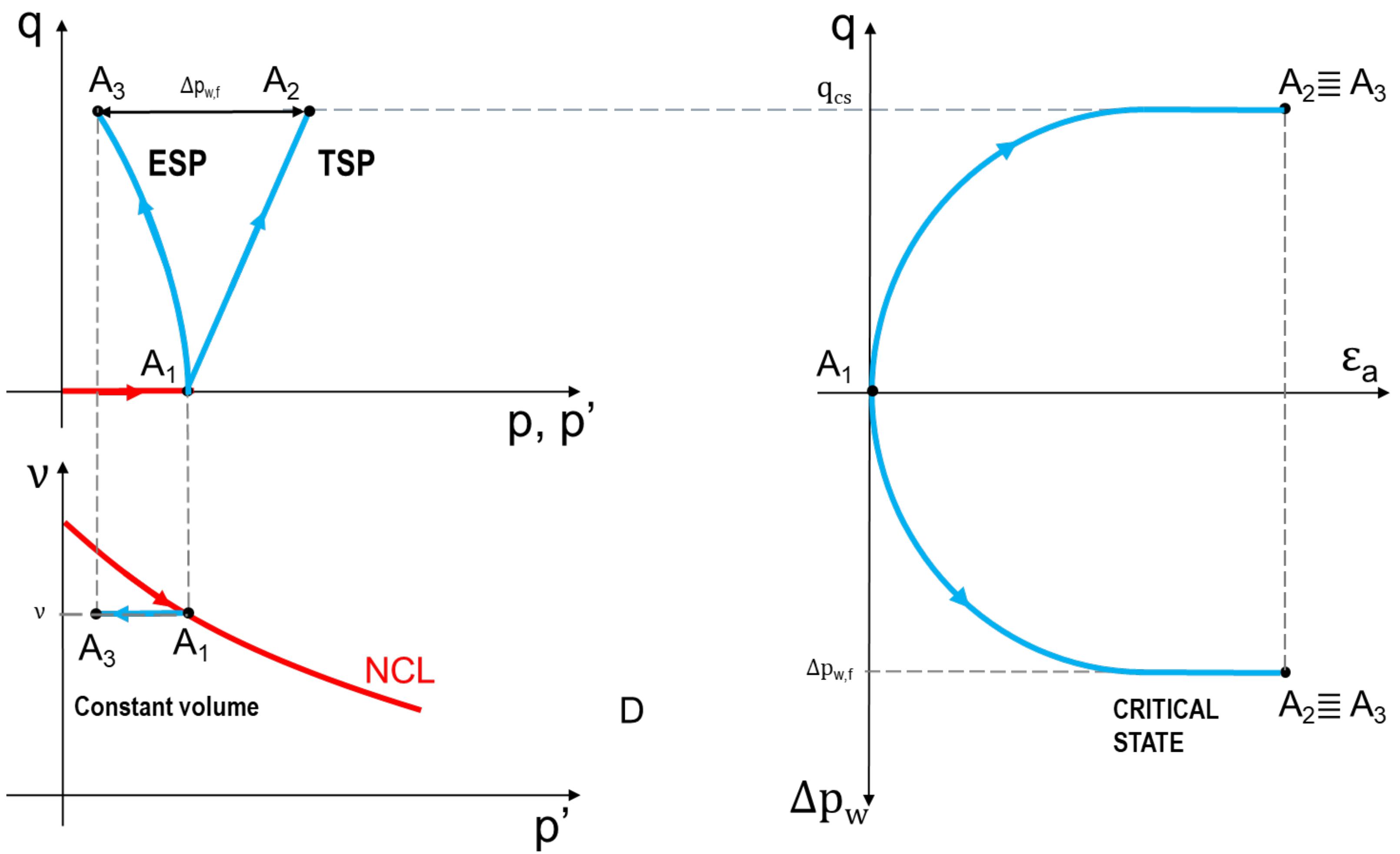
$$\Delta V = 0 \Rightarrow \varepsilon_v = 0$$

Shearing a NC soil in undrained compression

Stress path and stress-strain response

A single sample (A)

A_1 is the starting point of both the effective and total stress paths only if $p_{w,0}$=0



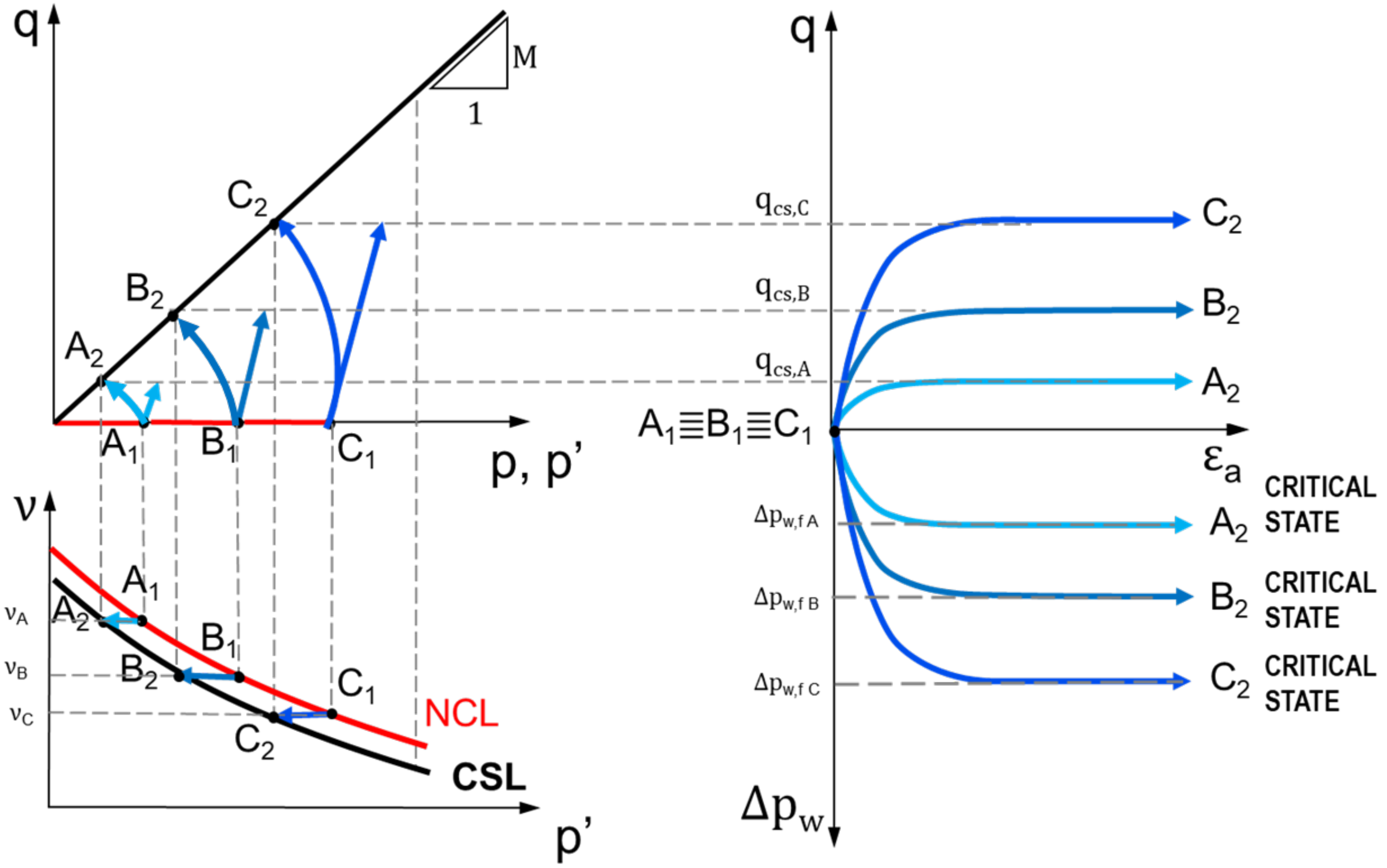
Shearing a NC soil in undrained compression

Stress path and stress-strain response

Three samples (A,B,C)

consolidated at three

different confinement



Shearing a NC soil

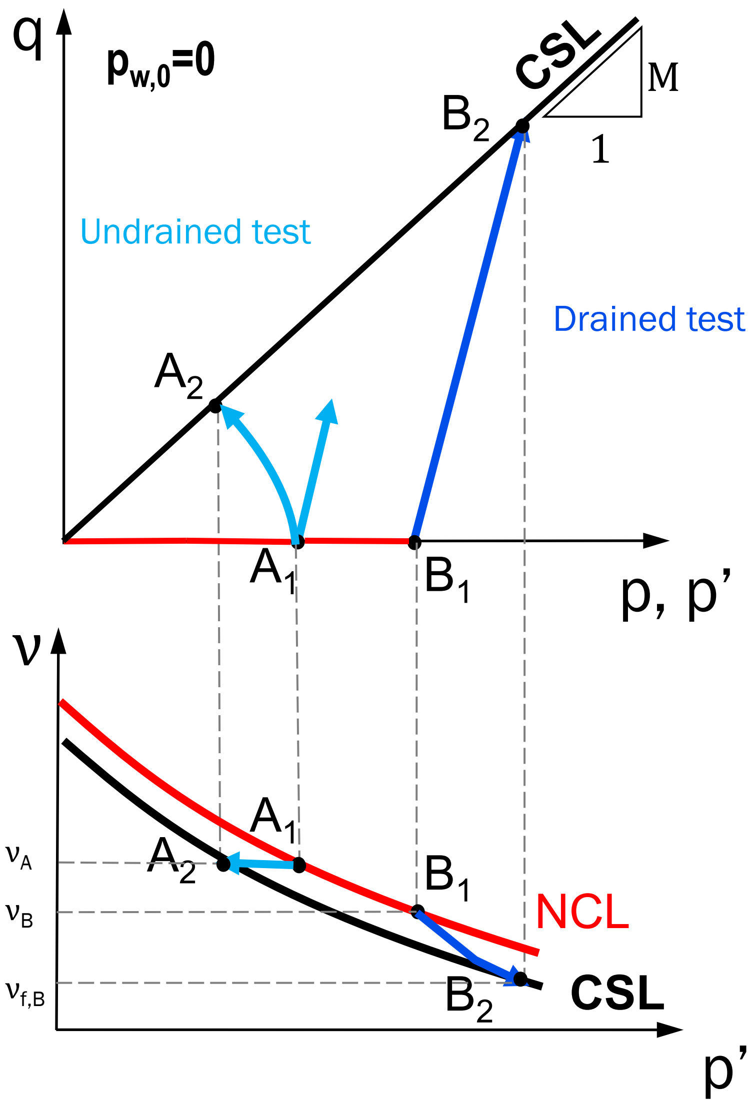
Critical State Lines in drained and undrained conditions

The **Critical State Lines** do not depend on the test conditions (drained or undrained)

$$\boxed{q = Mp'}$$

$$\boxed{\nu = \Gamma - \lambda \ln p'}$$

- If the soil is a NC clay (or a loose sand), the CSL is reached with a contracting behavior:
  - If V can change (**drain. cond.**)
    ➡ V decrease during shear (B_1-B_2)
  - If V cannot change (**undrain. cond.**)
    ➡ $\Delta p_w > 0$ during shear (A_1-A_2)



Critical State Line in q-p'-ν plane

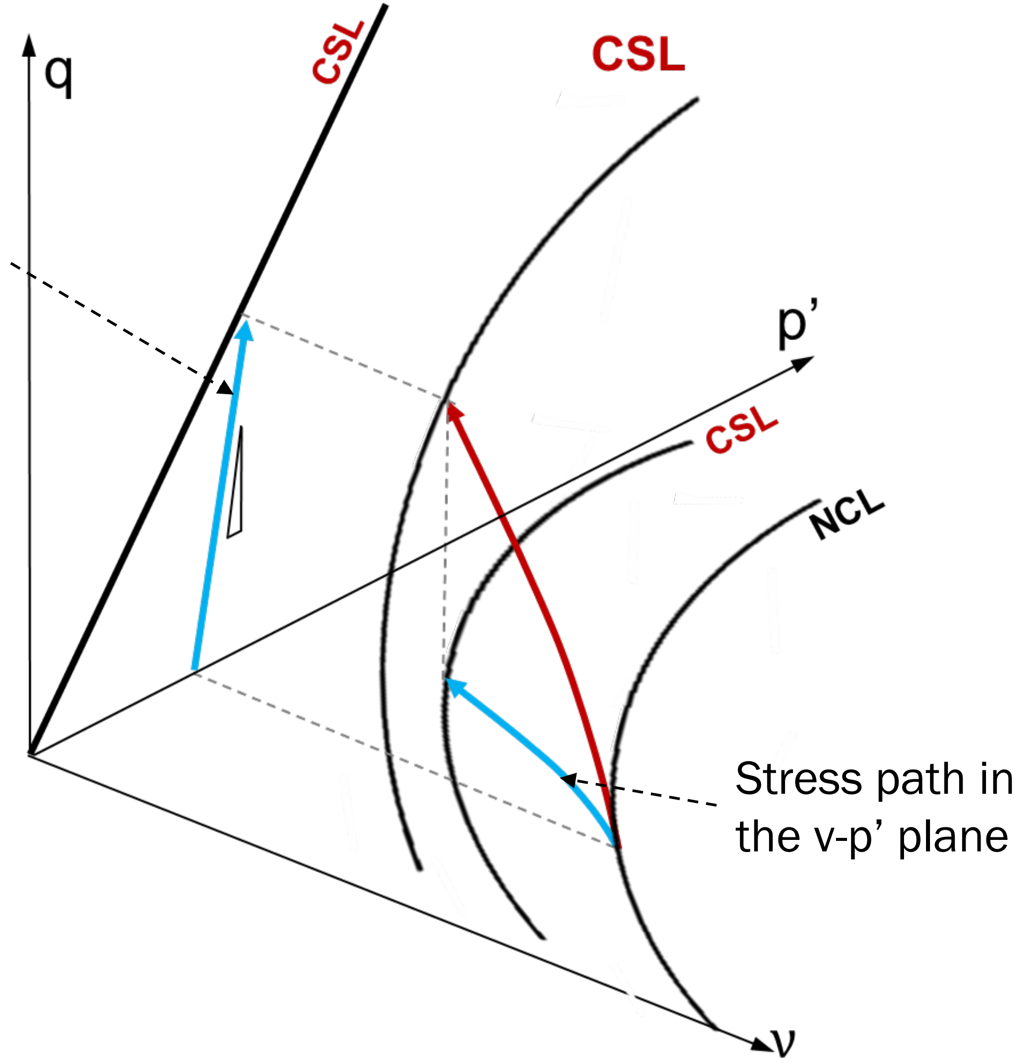
Combination of planes: Drained path

- **q - p'** plane

$$q = Mp'$$

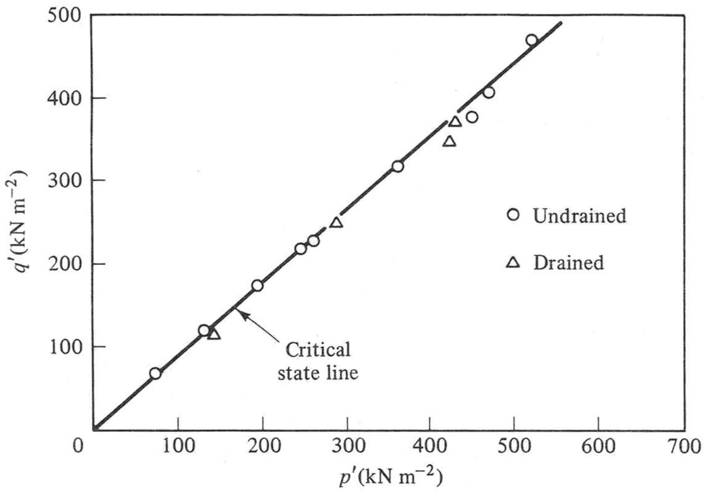
- **q - ν** plane

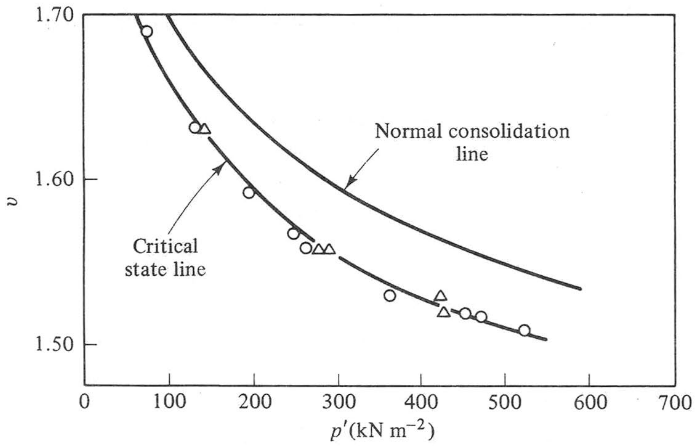
$$\nu = \Gamma - \lambda \ln p'$$

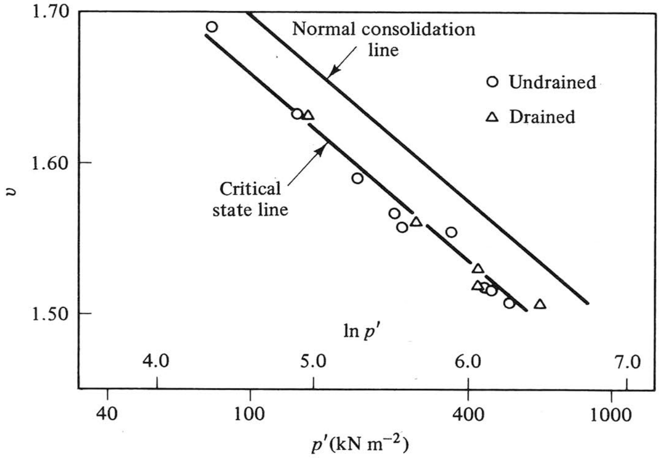


Critical state Line

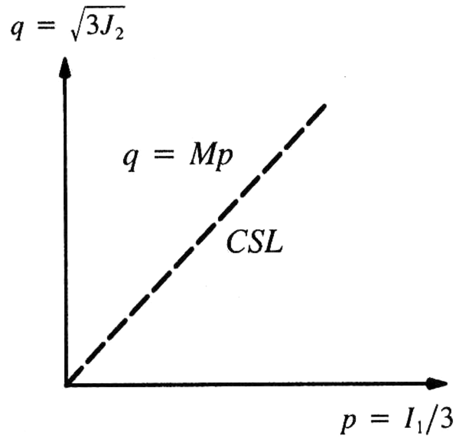
Combination of drained and undrained tests



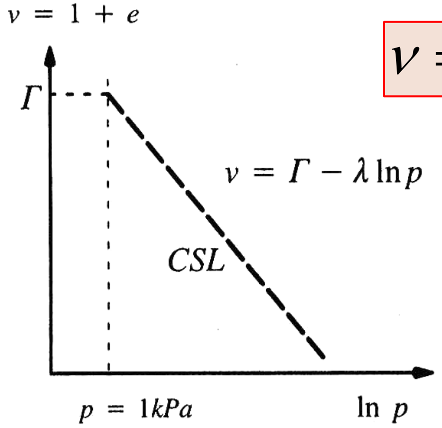




Critical state line



$$q = Mp'$$

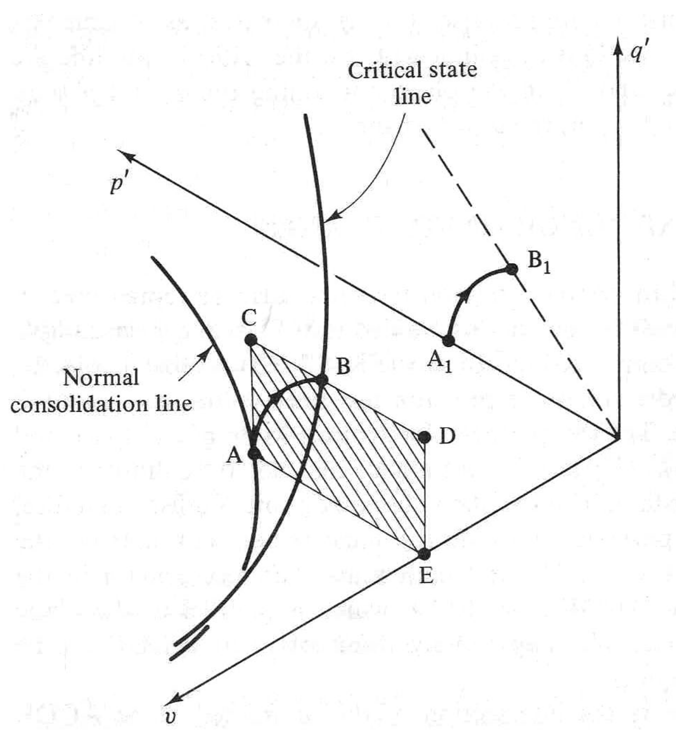


$$v = \Gamma - \lambda \ln p'$$

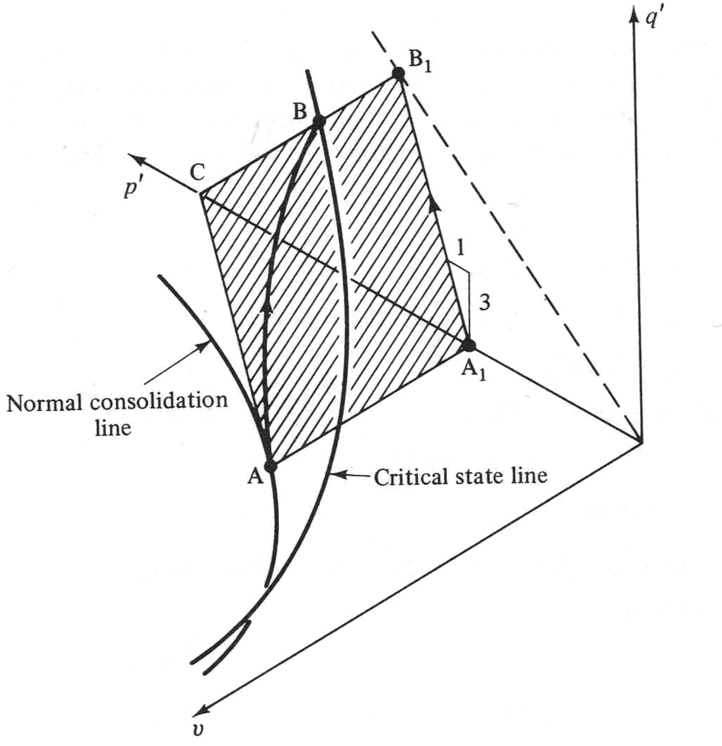
CSL: Drained & undrained path in q-p'-ν

Drained & Undrained: Triaxial CTC on normally consolidated samples until failure

Undrained



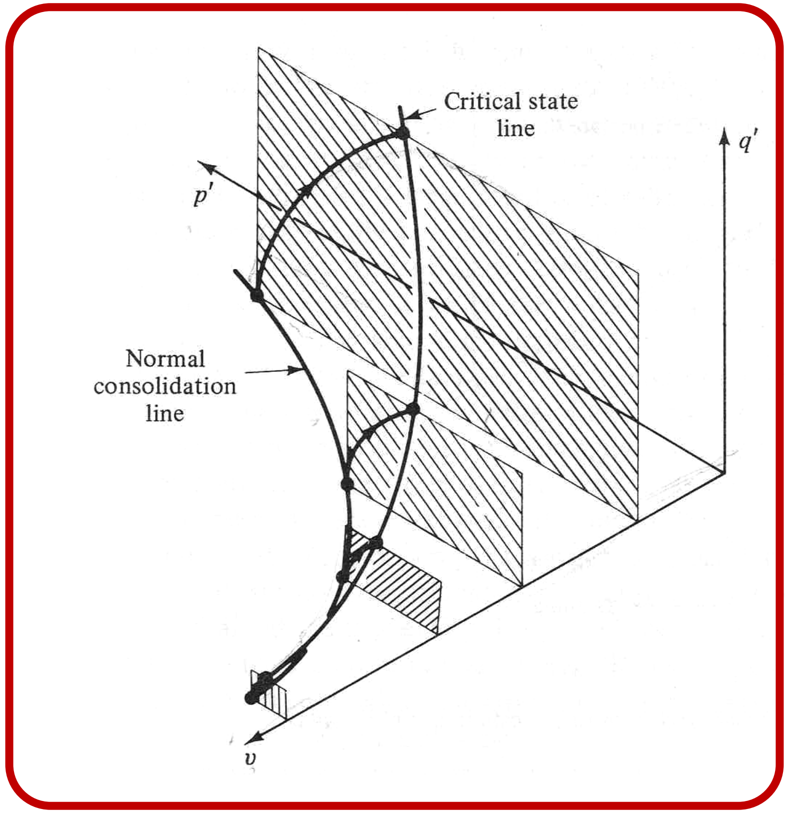
Drained



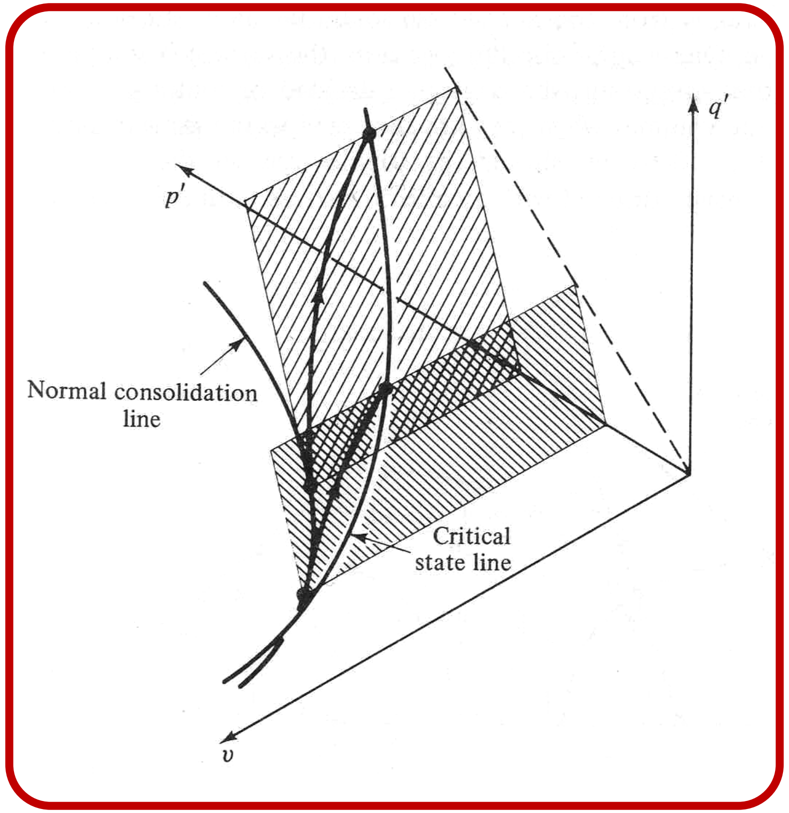
CSL: Families of drained & undrained tests in q-p'-ν

Drained & Undrained: Triaxial CTC on normally consolidated samples until failure

Undrained



Drained



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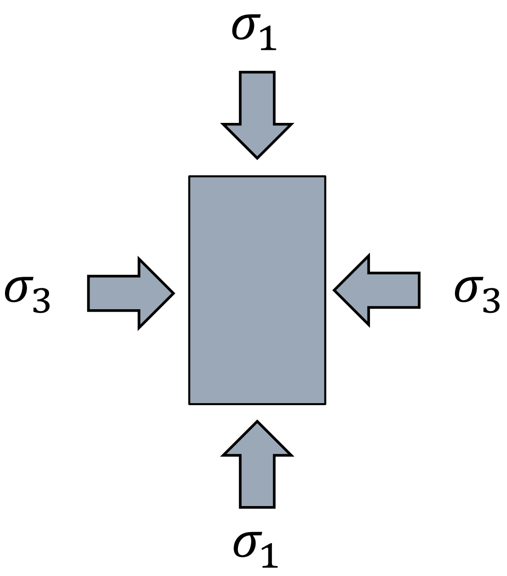
Shearing an OC soil in drained compression

Principle • Increase of the axial stress and imposed drained conditions

Use • Evaluation of the soil shear strength

Test • Second stage of a triaxial test with drainage valves open



Initial state of the soil $OCR = \frac{p'_0}{p'} > 1$

On the URL

Stress state variables

$$p = \frac{\sigma_1 + 2\sigma_3}{3} \qquad p' = \frac{\sigma'_1 + 2\sigma'_3}{3}$$

$$q = \sigma_1 - \sigma_3$$

Drained condition

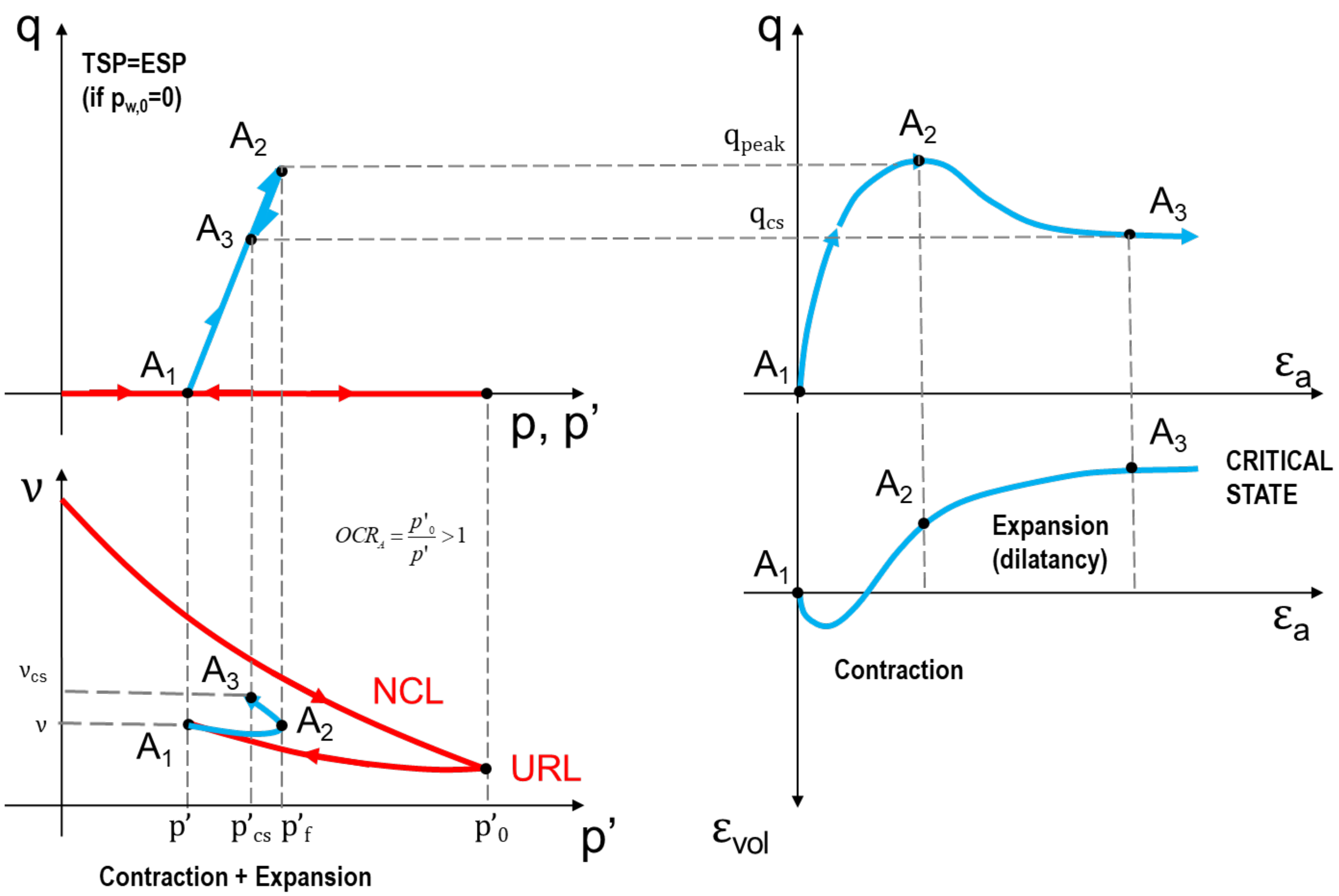
$$\Delta p_w = 0 \Rightarrow \Delta p = \Delta p'$$

If $p_{w,0} = 0 \Rightarrow p = p'$

Shearing an OC soil in drained compression

Stress path and stress-strain response

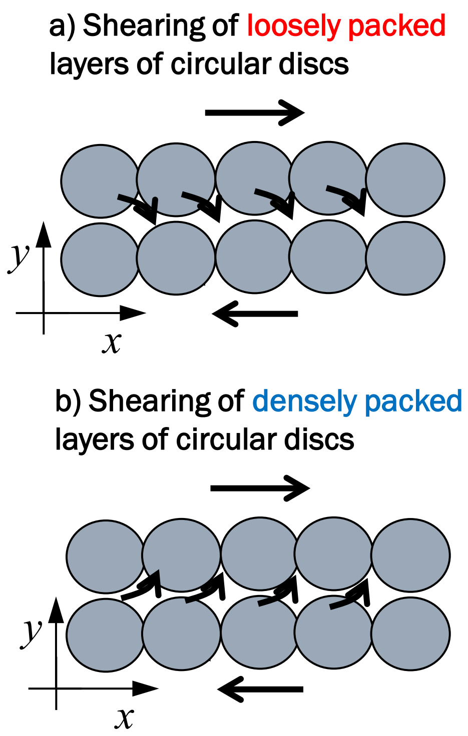
A single sample (A)

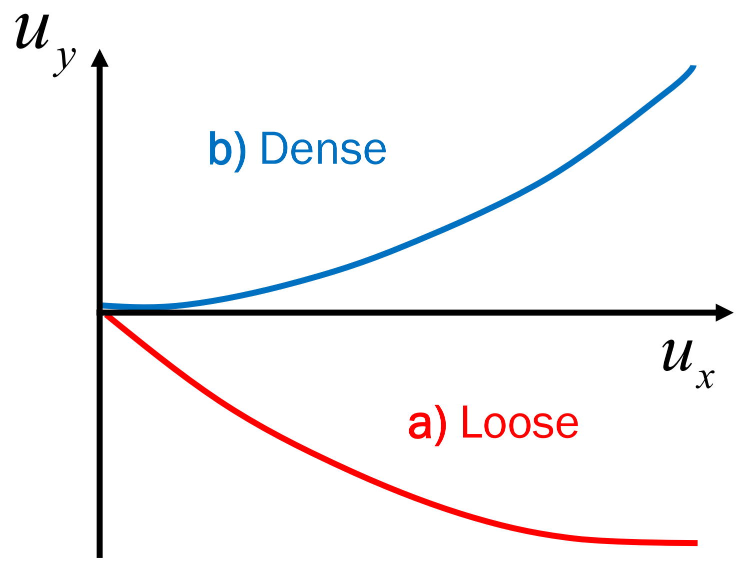


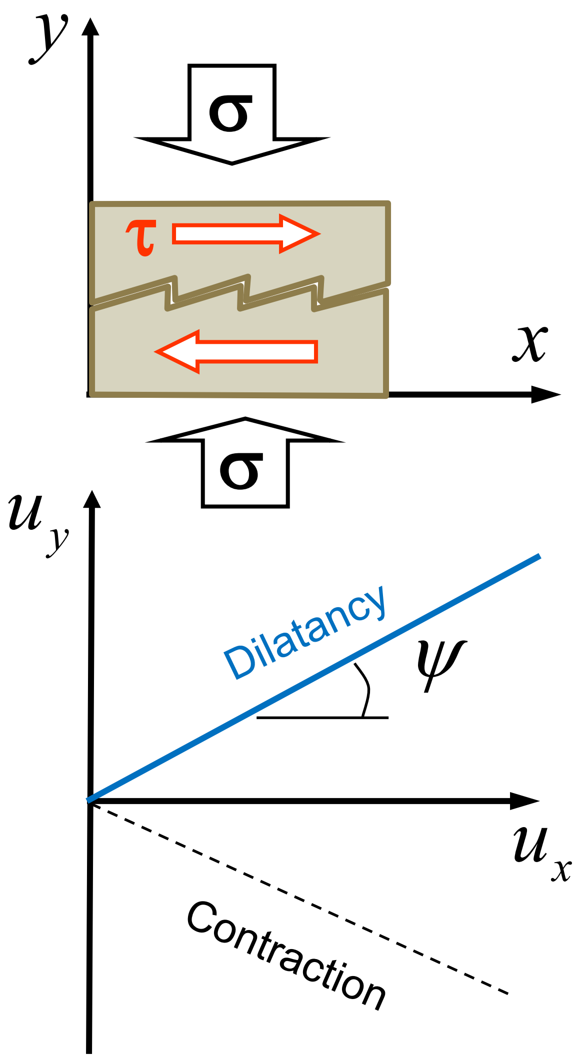
Shearing an OC soil in drained compression

Dilatancy and contractive behaviour

- The **density** defines the **contractive** or **dilating** behaviour of the soil

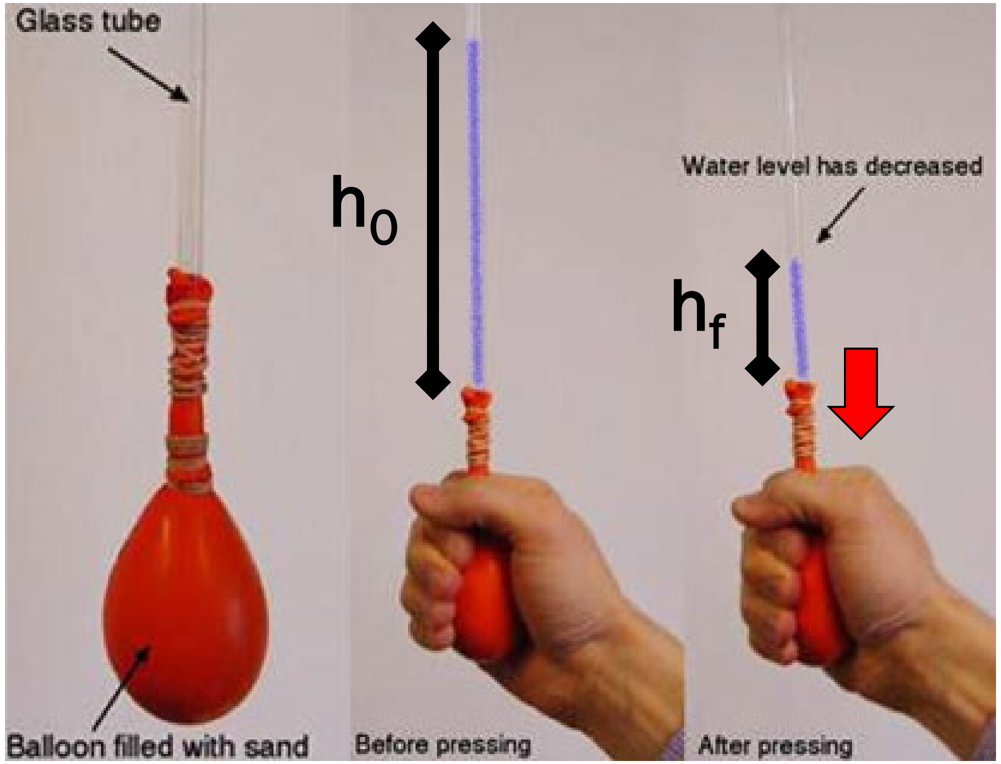






Shearing an OC soil in drained compression

Reynolds's experiments (1885) - Dilatancy



- A balloon is filled with dense sand.
- A glass tube is filled with water.

→ Pressing the balloon, water goes down on the tube.

Well compacted sand → tendency to expand

- the space between grains increase
- The upper water to enter within it.

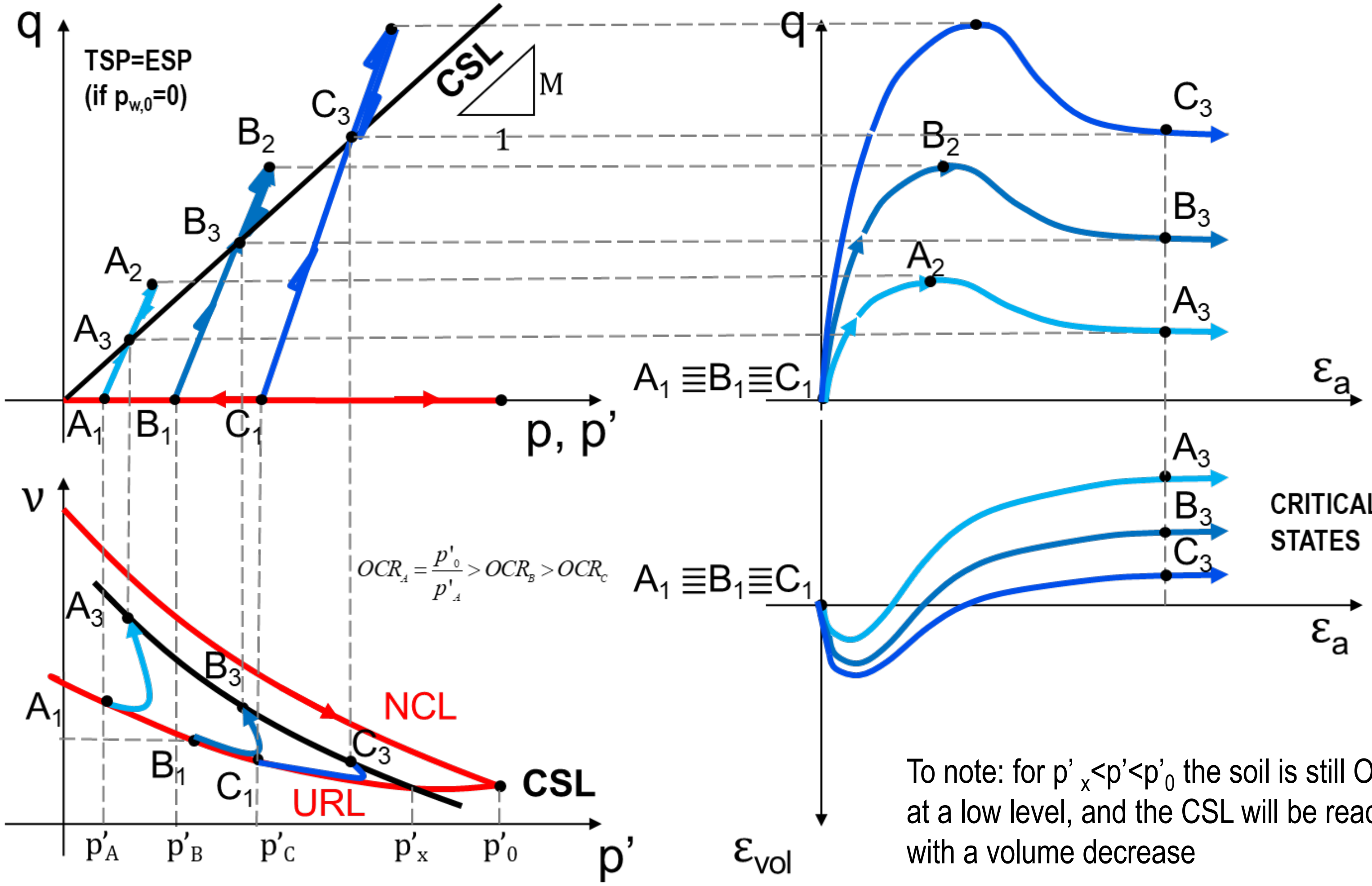
Shearing an OC soil in drained compression

Stress path and stress-strain response

Three samples (A,B,C)

consolidated at three

different confinement



The higher the OCR, the greater the volume increase that must occur for reaching the critical state (higher dilatancy)

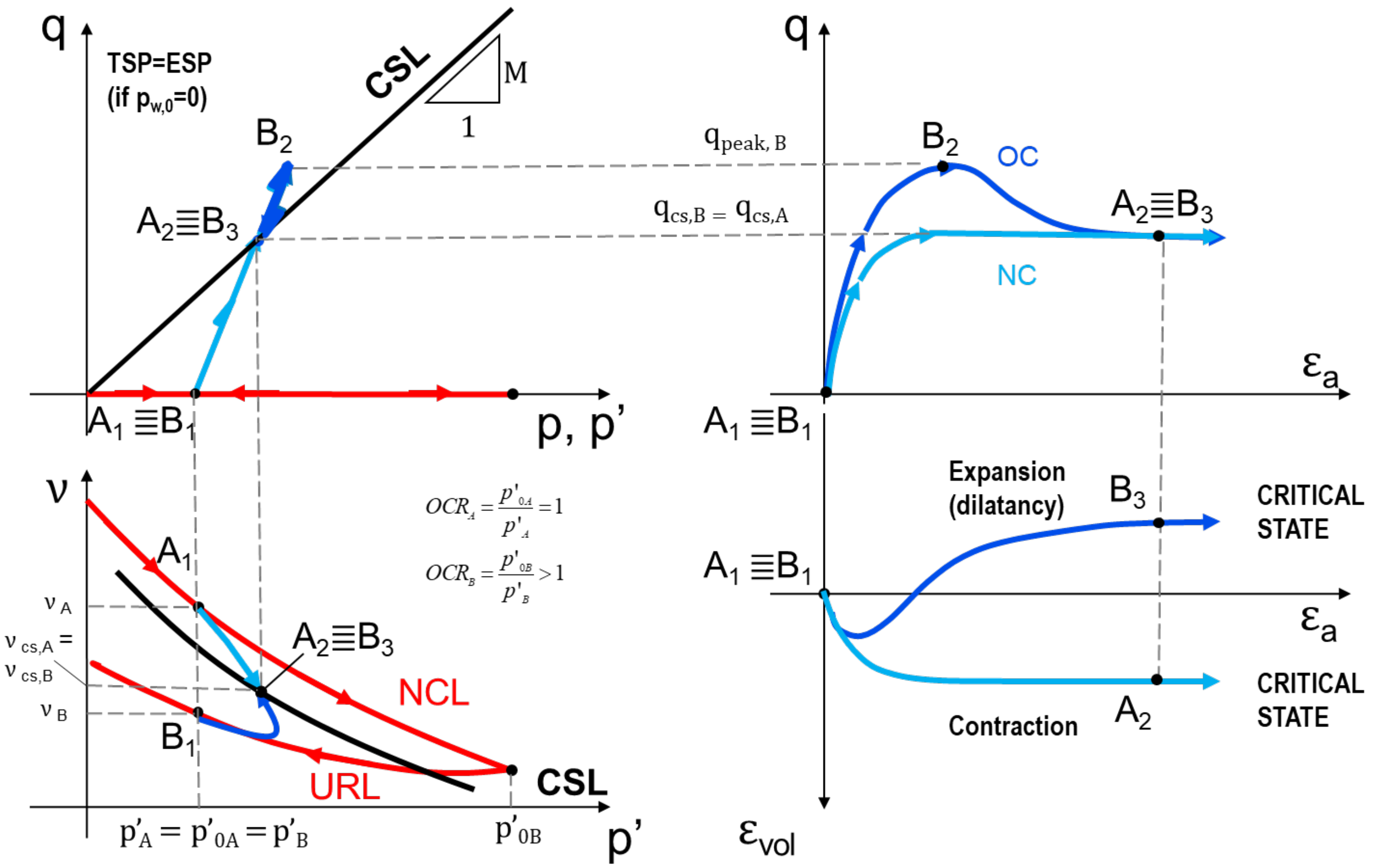
To note: for $p'_x < p' < p'_0$ the soil is still OC but at a low level, and the CSL will be reached with a volume decrease

Shearing in **drained** compression: NC and OC

Stress path and stress-strain response

Two different samples:

- A is NC
- B is OC



Shearing in **drained** compression: NC and OC

Critical State Line

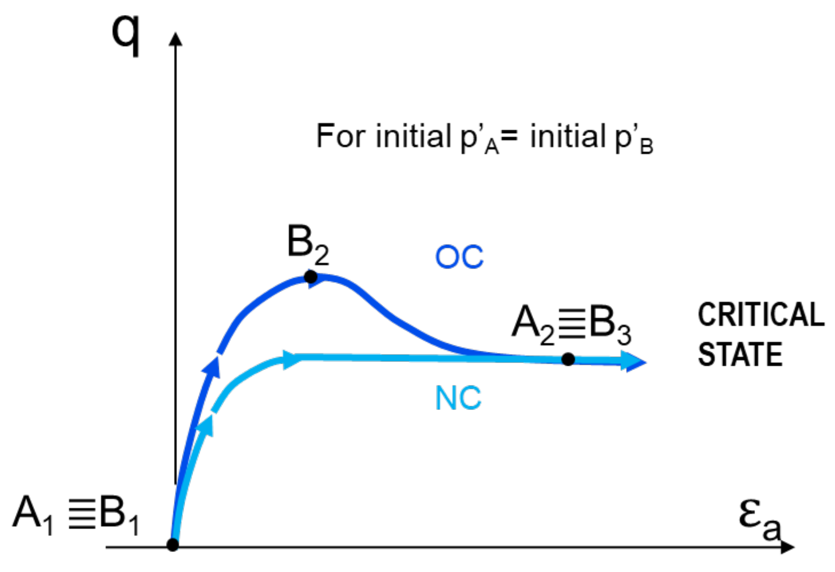
The **Critical State Lines** do not depend on the test conditions (NC or OC)

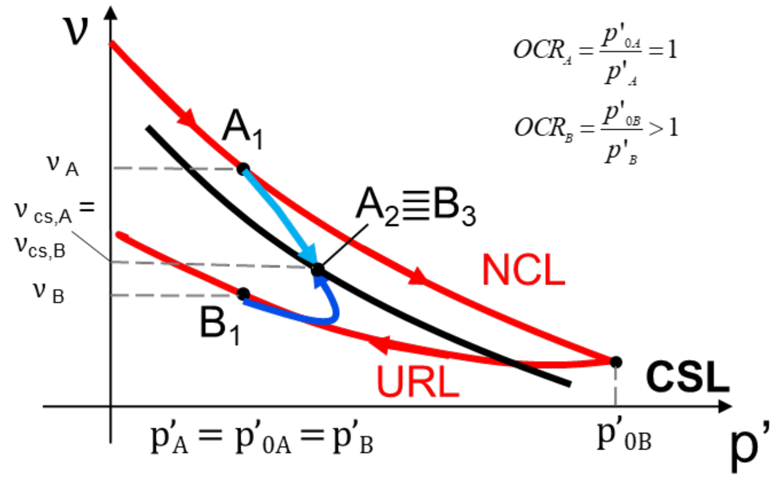
- In drained conditions the CSL is reached with a contracting behavior:
  - If the soil is a NC clay (or a loose sand) with a contractive behavior

    ➡ $\Delta V<0$ during shear (A_1-A_2)
  - If the soil is a OC clay (or a dense sand), with a dilatant behavior

    ➡ $\Delta V>0$ during shear (B_1-B_3)

    if the soil has a significant OCR (otherwise ΔV<0)



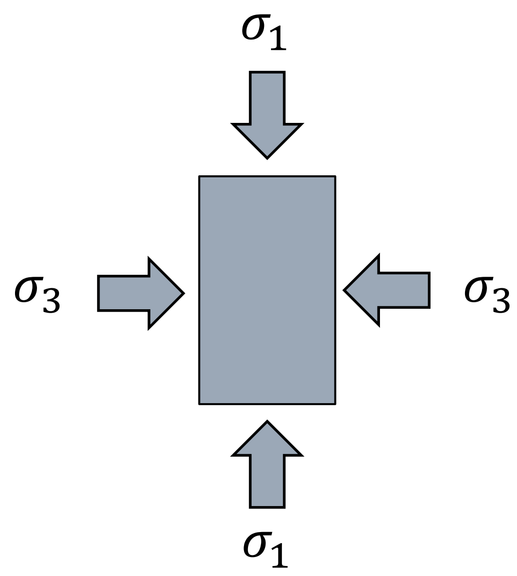


Shearing a OC soil in undrained compression

Principle
- Increase of the axial stress and imposed undrained conditions

Use
- Evaluation of the soil shear strength

Test
- Second stage of a triaxial test with drainage valves closed



Initial state of the soil $OCR = \frac{p'_0}{p'} > 1$

On the URL

Stress state variables

$$p = \frac{\sigma_1 + 2\sigma_3}{3} \qquad p' = \frac{\sigma'_1 + 2\sigma'_3}{3}$$

$$q = \sigma_1 - \sigma_3$$

Undrained condition

$$\Delta p_w \neq 0 \Rightarrow \Delta p \neq \Delta p'$$

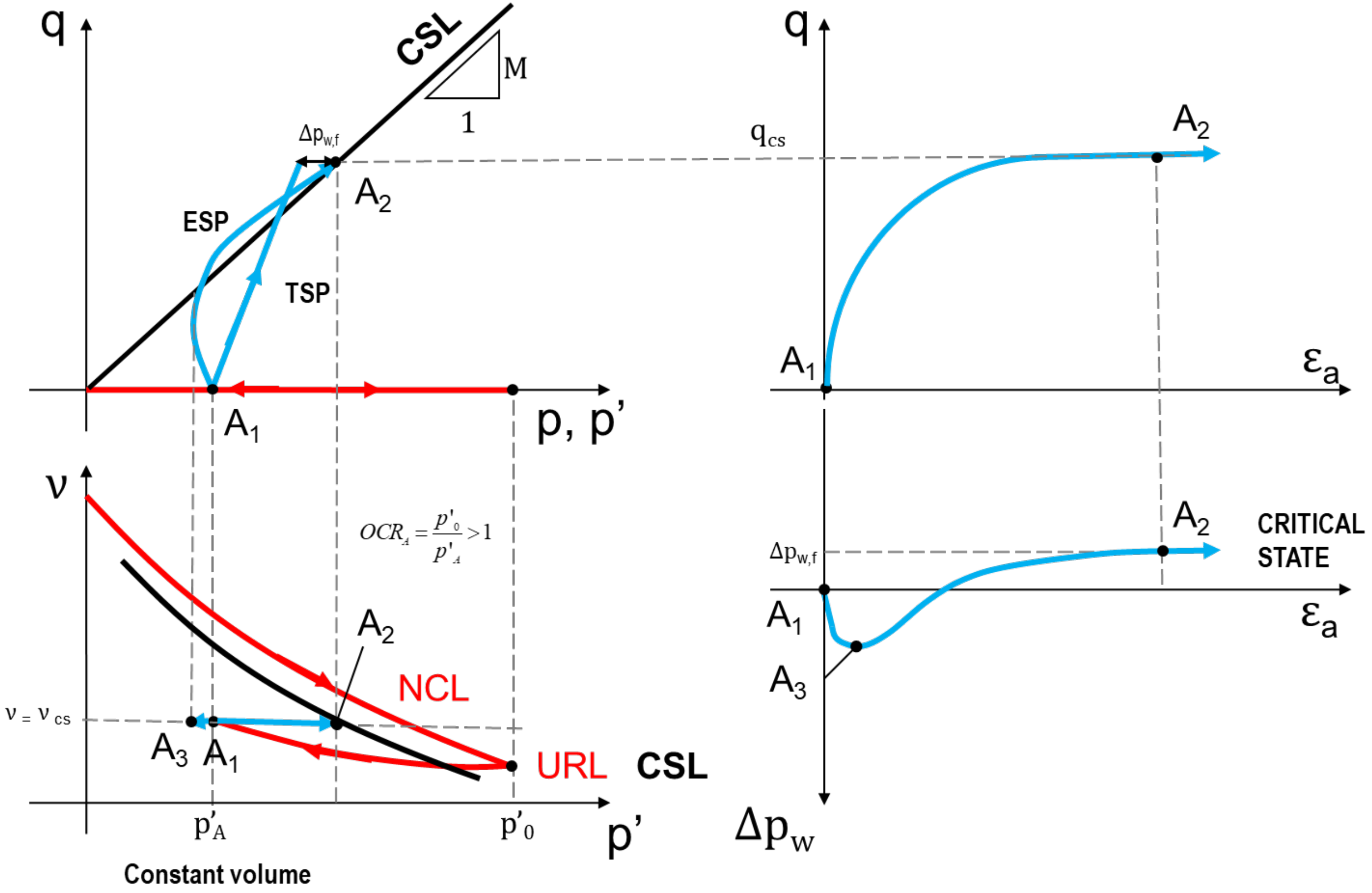
$$\Delta V = 0 \Rightarrow \varepsilon_v = 0$$

Shearing a OC soil in undrained compression

Stress path and stress-strain response

A single sample (A)

- The **dilating tendency** of a **OC** soil results in the development of a negative overpressure if the volume cannot change (undrained cond.)



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Critical state and shear strength

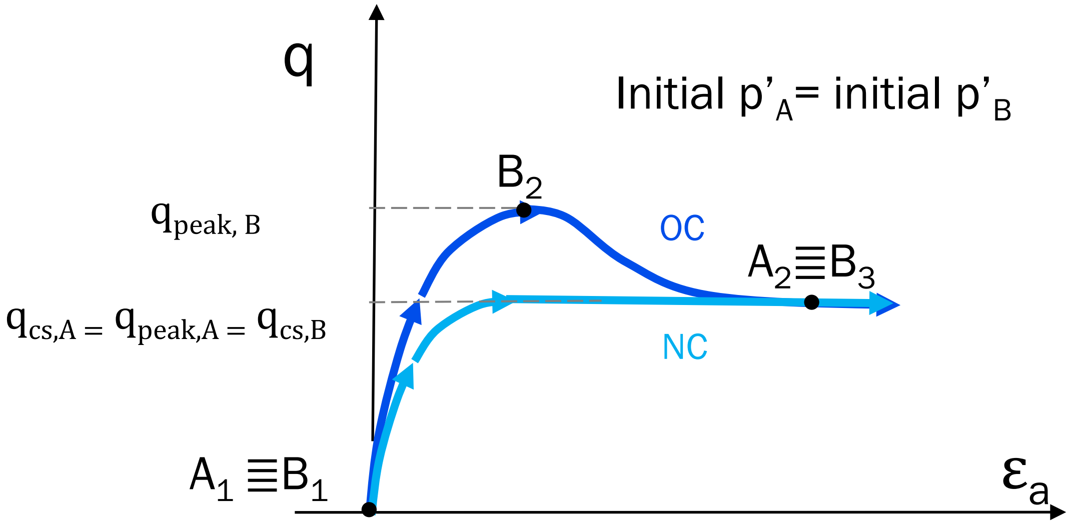
Shear strength definition

Shear strength → **Peak** shear strength (q_{peak}) / **Critical** (or ultimate) shear strength (q_{cs})

→ Only OC soil shows peak strength

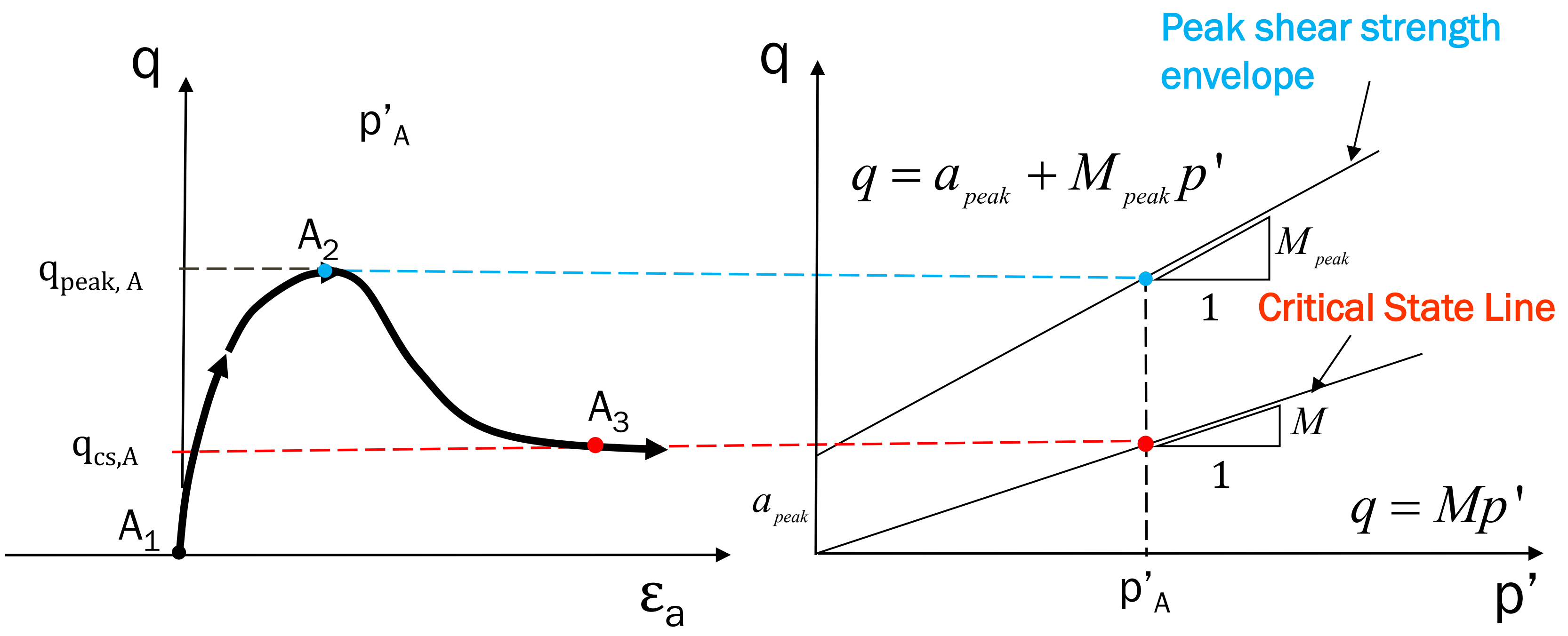
Two samples of the **same soil** consolidated at the **same p'** in **drained** conditions but **different OCRs:**

- A → NC $\quad q_{peak,A} = q_{cs,A}$
- B → OC $\quad q_{peak,B} \neq q_{cs,B}$



Critical state and shear strength

Soil Strength concept : Mohr-Coulomb failure law



→ **Soil strength** determined through the **Mohr-Coulomb failure law** correspond to the peak strength

→ **Soil strength** definition should be selected based on the engineering problem of interest

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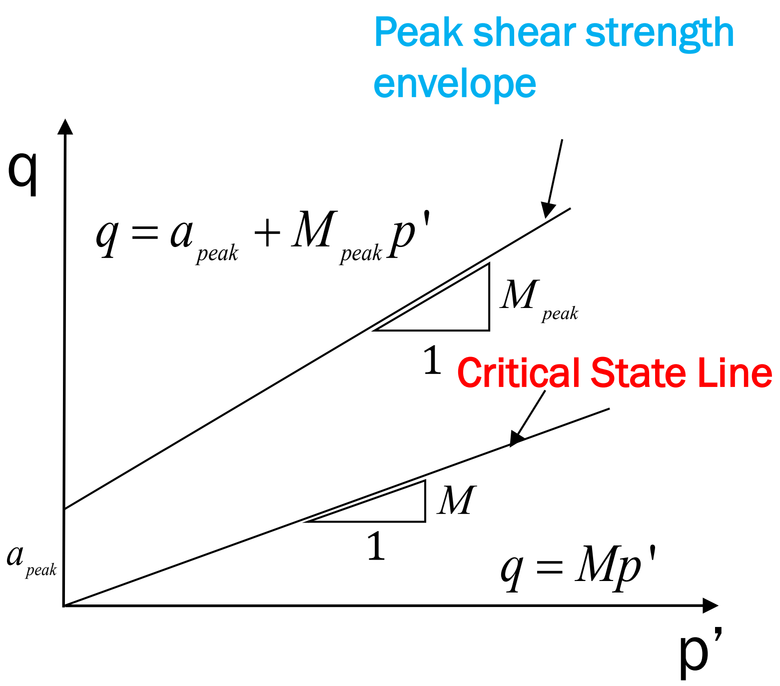
Soil Strength concept : Mohr-Coulomb failure law



→ Different samples with the **same initial density** are sheared at **different confinement stresses.**
The peak values are selected.

⇨ NOT UNIQUE (initial void ratio dependency)

$$q = a_{peak} + M_{peak} p'$$

→ Different samples are sheared at **different confinement stresses.**
The critical values are selected.

⇨ UNIQUE (no initial void ratio dependency)

$$q = Mp'$$

M : material parameter

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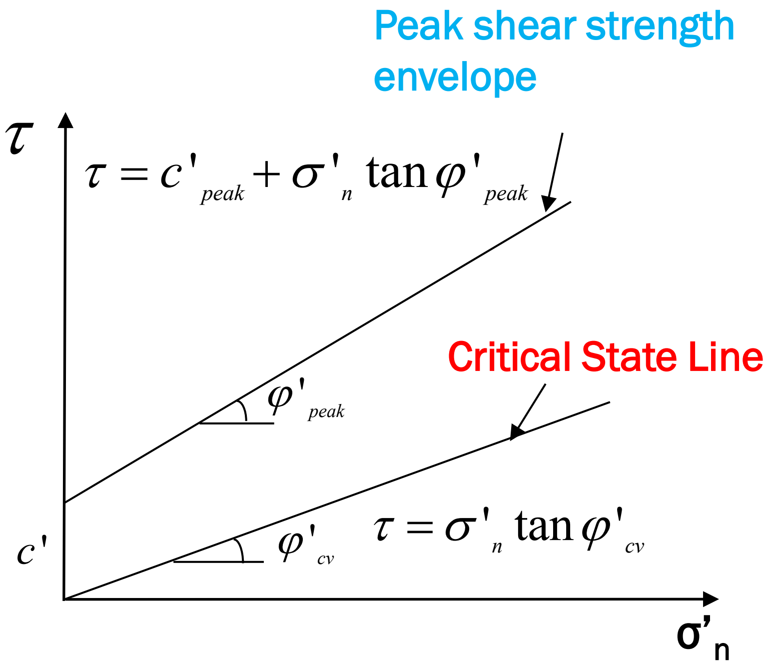
Soil Strength concept : Mohr-Coulomb failure law



→ From the q-p' plane to the τ - σ'_n plane:

$$\boxed{\tau = c'_{peak} + \sigma'_n \tan \varphi'_{peak}} \qquad \left[\begin{array}{l} \varphi'_{peak} = \arcsin\left(\dfrac{3M_{peak}}{6+M_{peak}}\right) \\ c'_{peak} = \dfrac{a_{peak}}{M_{peak}} \tan \varphi'_{peak} \end{array} \right.$$

→ From the q-p' plane to the τ - σ'_n plane:

$$\boxed{\tau = \sigma'_n \tan \varphi'_{cv}} \qquad \varphi'_{cv} = \arcsin\left(\frac{3M}{6+M}\right)$$

Content

- The critical state concept
- Experimental evidences Conventional Triaxial Compression tests
  - Isotropic drained compression
  - Shearing phase on a NC soil in drained and undrained conditions
  - Shearing phase on a OC soil in drained and undrained conditions
- Critical state and shear strength
- Examples

Example

For a given soil, the critical state line is defined by the following soil parameters:

$$M = 0.9, \lambda = 0.25, \Gamma = 3.0$$

The soil was subjected to conventional drained and undrained triaxial compression tests, calculate the values of q, p' and v at the critical state as well as the pore pressure for the undrained test when the soil initial condition is:

a) $p_i' = 100\ kPa, q_i = 0\ kPa, v_i = 2.0$

b) $p_i' = 100\ kPa, q_i = 0\ kPa, v_i = 1.7$

Example (a) – Drained loading

Soil parameters: $M = 0.9, \lambda = 0.25, \Gamma = 3.0$

a) $p_i' = 100\ kPa, q_i = 0\ kPa, v_i = 2.0$

$\Delta q = 3 \cdot \Delta p'\ ,\ q_{cs} = M \cdot p_{cs}'$

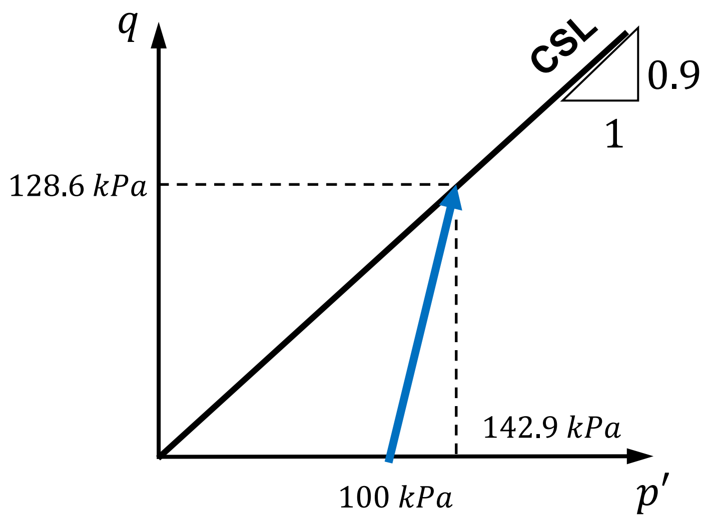
Drained loading → ESP = TSP → $\Delta q = q_{cs} = 0.9 \cdot p_{cs}' = 3 \cdot \Delta p'$

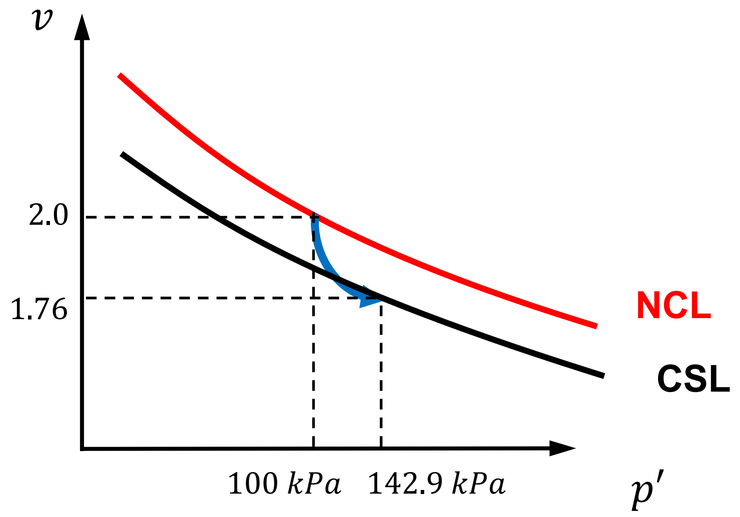
$p_{cs}' = p_i' + \Delta p'$

$0.9p_i' + 0.9\Delta p' = 3\Delta p'$ → $\Delta p' = 42.9\ kPa$

$p_{cs}' = 100 + 42.9 = 142.9\ kPa\ ,\quad q_{cs} = 0.9 \cdot 142.9 = 128.6\ kPa$

$v_{cs} = \Gamma - \lambda \cdot \ln(p_{cs}') = 3.0 - 0.25 \cdot \ln(142.9) = 1.76$





Example (a) – Undrained loading

Soil parameters: $M = 0.9, \lambda = 0.25, \Gamma = 3.0$

a) $p'_i = 100\ kPa, q_i = 0\ kPa, v_i = 2.0$

$\Delta q = 3 \cdot \Delta p'\ ,\ q_{cs} = M \cdot p'_{cs}$

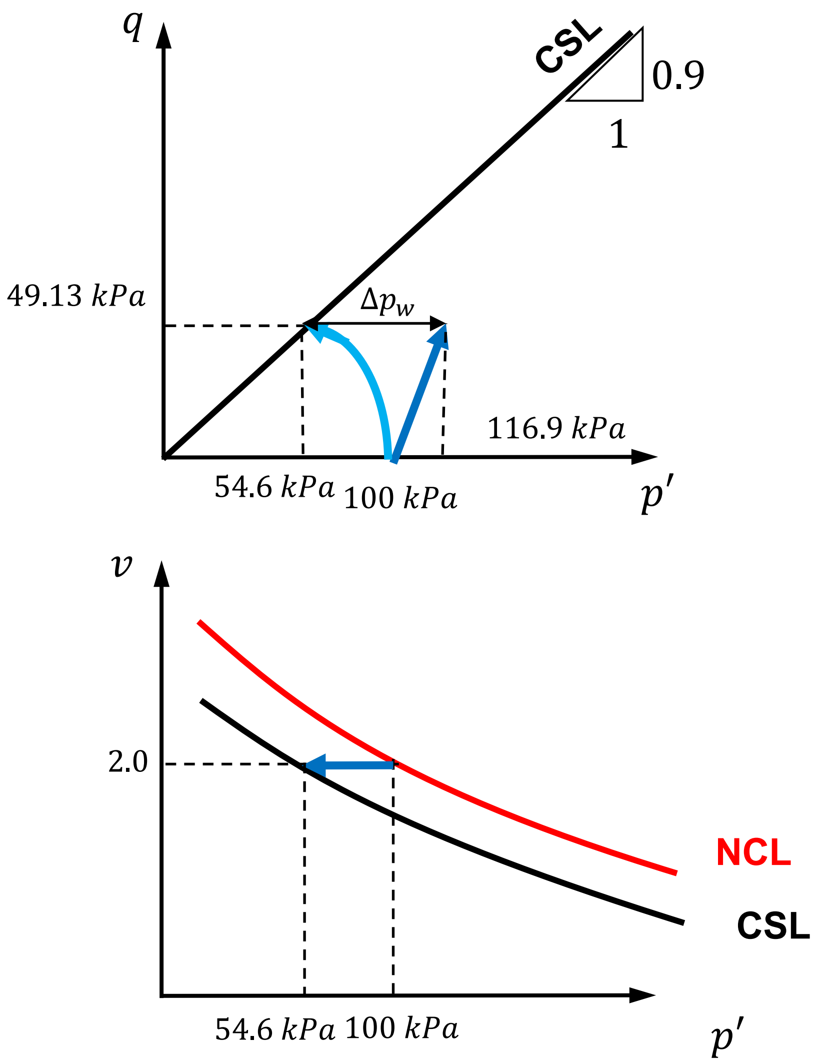
Undrained → $\varepsilon_v = 0$ → $v_{cs} = v_i = \Gamma - \lambda \cdot \ln(p'_{cs})$

$p'_{cs} = \exp\left(\frac{\Gamma - v}{\lambda}\right) = \exp(4) = 54.6\ kPa$

$q_{cs} = 0.9 \cdot 54.6 = 49.13\ kPa$

$\Delta q = q_{cs} = 3 \cdot \Delta p$ → $p = \Delta p + p_i = \frac{49.13}{3} + 100 = 116.4\ kPa$

$p_w = p - p' = 61.7\ kPa$



Example (b) – Drained loading

Soil parameters: $M = 0.9, \lambda = 0.25, \Gamma = 3.0$

b) $p'_i = 100\ kPa, q_i = 0\ kPa, v_i = 1.7$

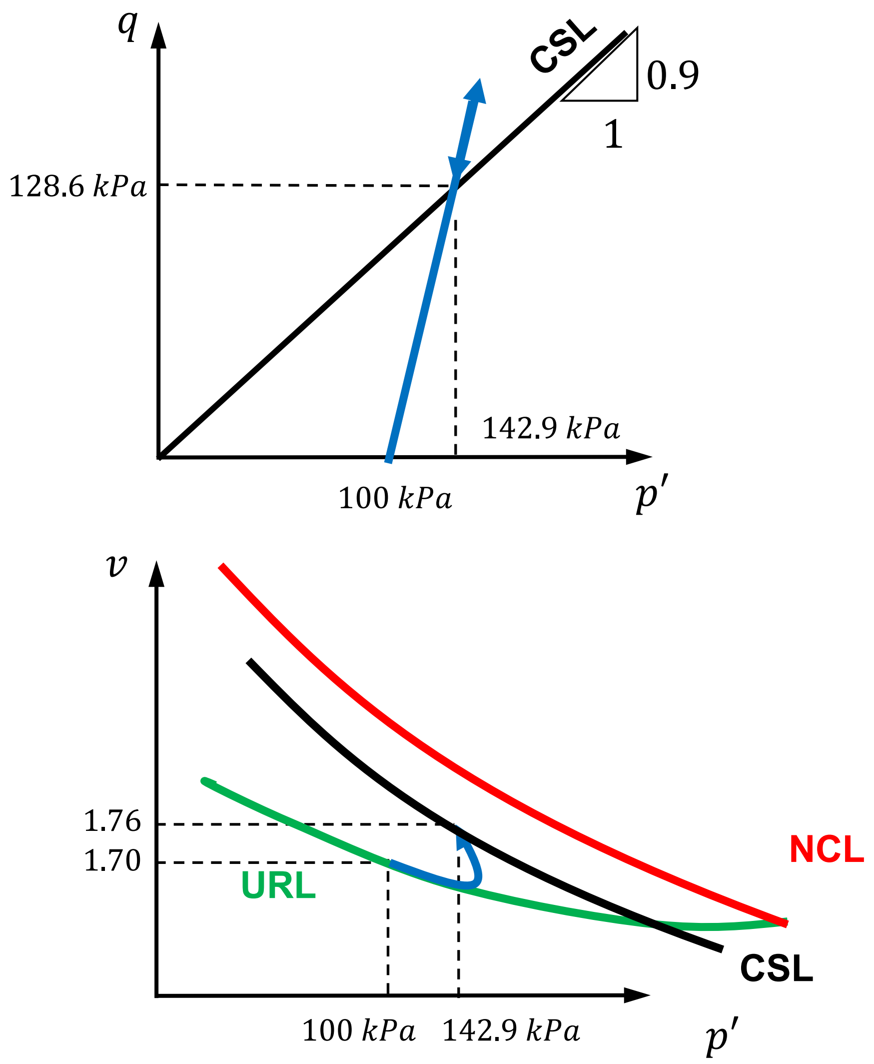
$\Delta q = 3 \cdot \Delta p'\ ,\ \ q_{cs} = M \cdot p'_{cs}$

Drained loading → ESP = TSP →$\Delta q = q_{cs} = 0.9 \cdot p'_{cs} = 3 \cdot \Delta p'$

$p'_{cs} = p'_i + \Delta p'$→ $0.9p'_i + 0.9\Delta p' = 3\Delta p'$ → $\Delta p' = 42.9\ kPa$

$p'_{cs} = 142.9\ kPa\ , q_{cs} = 128.6\ kPa$

$v_{cs} = \Gamma - \lambda \cdot \ln(p'_{cs}) = 1.76$



Example (b) – Undrained loading

Soil parameters: $M = 0.9, \lambda = 0.25, \Gamma = 3.0$

b) $p'_i = 100\ kPa, q_i = 0\ kPa, v_i = 1.7$

$\Delta q = 3 \cdot \Delta p'\ ,\ q_{cs} = M \cdot p'_{cs}$

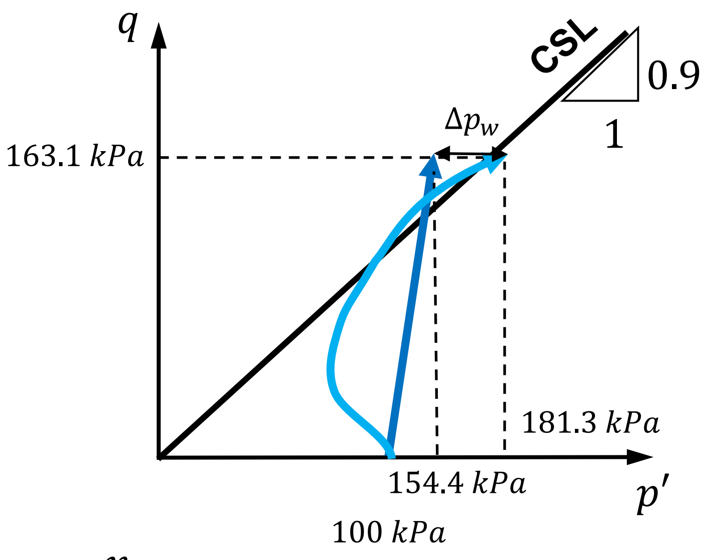
Undrained → $\varepsilon_v = 0$ → $v_{cs} = v_i = \Gamma - \lambda \cdot \ln(p'_{cs})$

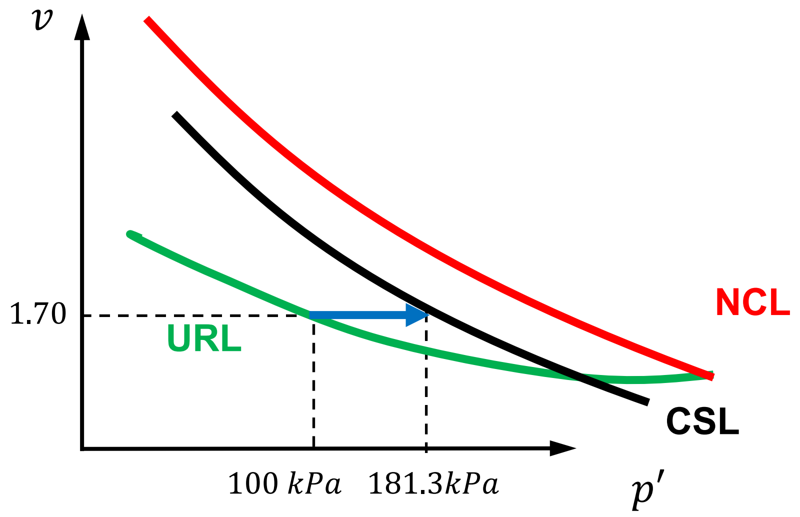
$p'_{cs} = \exp\left(\frac{\Gamma - v}{\lambda}\right) = \exp(5.2) = 181.3\ kPa$

$q_{cs} = 0.9 \cdot 181.3 = 163.14\ kPa$

$\Delta q = q_{cs} = 3 \cdot \Delta p$ → $p = \Delta p + p_i = \frac{163.14}{3} + 100 = 154.38\ kPa,$

$p_w = p\ - p' = -26.9\ kPa$





Summary

- The critical state is a state in which the shearing can continue without any tendency of the soil to change its volume. It corresponds to reaching a constant volume condition in the case of a drained test, and to reaching a constant water pressure condition in the case of an undrained test.
- The critical state represents a lower bound to the strength of soils.
- Critical State Line (CSL) is defined in the $q - p' - \nu$ space
- CSL does not depend on the test conditions (drained or undrained)
- CSL does not depend on the state of the geomaterial (NC or OC)
- The critical state parameters are material parameters of the tested geomaterial (Γ, λ and M)
- The CSL of the q-p' plane can also be represented in the τ-σ'_n plane (critical or ultimate shear strength envelope) and it corresponds to a Mohr-Coulomb failure law with $c'=0$ and φ'_{cv}

Thank you for your attention